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Editor : Anil Ahlawat

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Creation of Antimatter

While the antiparticles were discovered in the last century, thanks to the powerful mass spectrometers and nuclear reaction studies.

The rules of formation of antimatter is not much different from that of matter. The major difficulty in formation is that, as the earth is full of matter, the slightest lack of isolation, matter and antimatter annihilate each other.

Physicists from CERN, where incidentally they have made one of the most powerful accelerators in the world, had isolated antihydrogen nuclei i.e. antiprotons. These are of sufficient low energies to enable experiments to be performed. When antielectrons are made to combine with them, antihydrogen atoms are generated. Physicists from CERN's Atomic Spectroscopy And Collisions Using Slow Antiprotons (ASACUSA) have produced at least 80 atoms of antihydrogen. This is a very important step for the march of science.

Spectroscopy had advanced by leaps and bounds in the last century. Atomic, molecular, X-ray, γ -ray spectroscopy, mass spectroscopy etc. had advanced not only experimentally but had also made immense contribution to atomic and molecular physics. To explain these spectra, theoretical advances were made in field theory and quantum mechanics. If scientists will reopen these chapters to study the spectroscopy of antimatter in every field, it will be really great. Knowing human nature, we only pray to God that they will not try to misuse the knowledge for destruction.

However optimism and hope is the solution for human existence.

Anil Ahlawat
Editor

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Physics Musing was started in August 2013 issue of Physics For You with the suggestion of Shri Mahabir Singh. The aim of Physics Musing is to augment the chances of bright students preparing for JEE (Main and Advanced) / AIPMT / AIIMS / Other PMTs / PETs with additional study material.

In every issue of Physics For You, 10 challenging problems are proposed in various topics of JEE (Main and Advanced) / AIPMT. The detailed solutions of these problems will be published in next issue of Physics For You.

The readers who have solved five or more problems may send their solutions along with address. The names of those who send atleast five correct solutions will be published in the next issue.

We hope that our readers will enrich their problem solving skills through "Physics Musing" and stand in better stead while facing the competitive exams.

By : Akhil Tewari

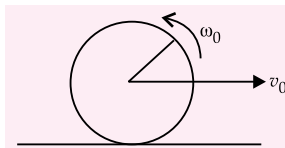
PROBLEM Set 8

1. A particle executes simple harmonic motion along a straight line with mean position at $x = 0$, period 20 s and amplitude 5 cm. The (shortest) time in seconds taken by the particle to go from $x = 4$ cm to $x = -3$ cm is

(a) 5 (b) 6
(c) 7 (d) 4

2. A uniform circular disc of radius r is placed on a rough horizontal surface and given a linear velocity v_0 and angular velocity ω_0 as shown. The disc comes to rest after moving some distance to the right. It follows that

(a) $v_0 = \omega_0 r$ (b) $2v_0 = \omega_0 r$
(c) $2v_0 = 3\omega_0 r$ (d) $3v_0 = 2\omega_0 r$



3. A substance of mass M kg requires a power input of P W (joules per second) to remain in the molten state at its melting point. When the power source is turned off, the sample completely solidifies in t s. The latent heat of fusion of the substance is

(a) $\frac{P}{Mt}$ (b) $\frac{Mt}{P}$ (c) $\frac{Pt}{2M}$ (d) $\frac{Pt}{M}$

4. A thin plastic disc of inner radius R_1 and outer radius R_2 has a charge q uniformly distributed over its surface. If the disc rotates at an angular frequency ω about the axis passing through its centre and perpendicular to its plane, the magnetic field at the centre of the disc is

(a) $\frac{\mu_0 \omega q}{2\pi R_1}$ (b) $\frac{\mu_0 \omega q}{2\pi(R_1 + R_2)}$

(c) $\frac{\mu_0 \omega q}{2\pi(R_2 - R_1)}$ (d) $\frac{\mu_0 \omega q}{2\pi(R_1 - R_2)}$

5. White light is used to illuminate the two slits in a Young's double slit experiment. The separation between slits is b and the screen is at a distance d ($\gg b$) from the slits. At a point on the screen directly in front of one of the slits, certain wavelengths are missing. One of these missing wavelengths is

(a) $\lambda = \frac{b^2}{2d}$ (b) $\lambda = \frac{2b^2}{d}$
(c) $\lambda = \frac{b^2}{3d}$ (d) $\lambda = \frac{2b^2}{3d}$

6. Which of the following statements is true concerning the elastic collision of two objects?

(a) No work is done on any of the two objects, since there is no external force.
(b) The work done by the first object on the second is equal to the work done on the first by the second.
(c) The work done by the first object on the second is exactly the negative of the work done on the first by the second.
(d) The work done on the system depends on the angle of collision.

7. A perfectly absorbing, black, solid sphere with constant density and radius R , hovers stationary above the sun. This is because the gravitational attraction of the sun is balanced by the pressure due to sun's light. Light pressure P is given by the intensity I of the absorbed light divided by the speed of light $c = 3 \times 10^8$ m s⁻¹. ($P = I/c$). Assume that the sun is far enough away that it closely approximates a point source of light. The distance

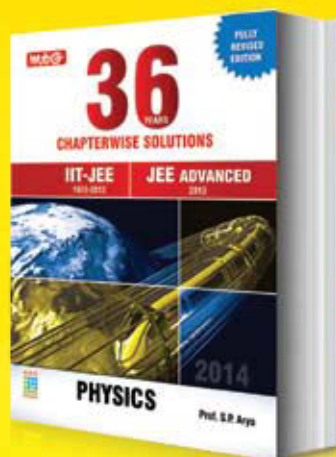
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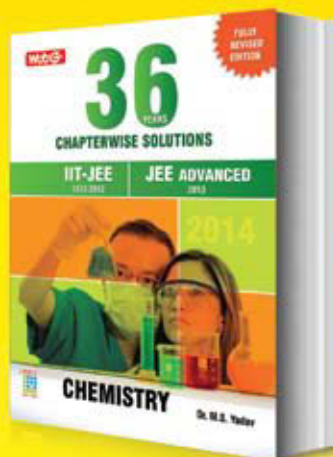
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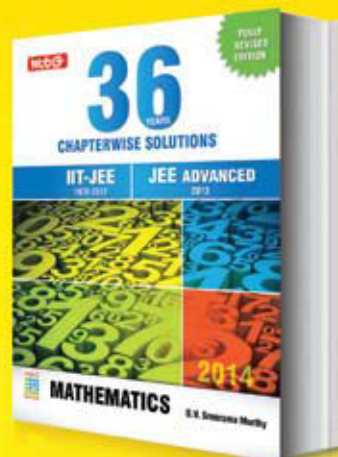
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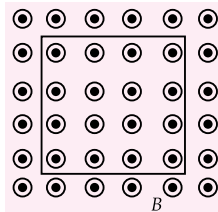


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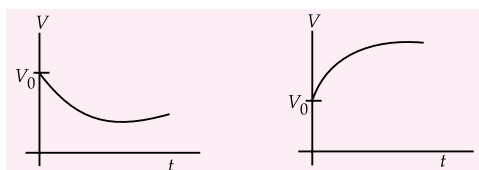
from the centre of the sun at which the sphere hovers is

- (a) proportional to R (b) proportional to $1/R$
(c) proportional to $1/R^2$ (d) independent of R

- 8 A uniform magnetic field B is directed out of the page. A metallic wire has the shape of a square frame and is placed in the field as shown. While the shape of the wire is steadily transformed into a circle in the same plane, the current in the frame



- (a) is directed clockwise
(b) does not appear
(c) is directed counterclockwise
(d) is alternating
9. A capacitor is charged upto a potential V_0 . It is then connected to a resistance R and a battery of emf E . Two possible graphs of potential across capacitor *vs* time are shown.

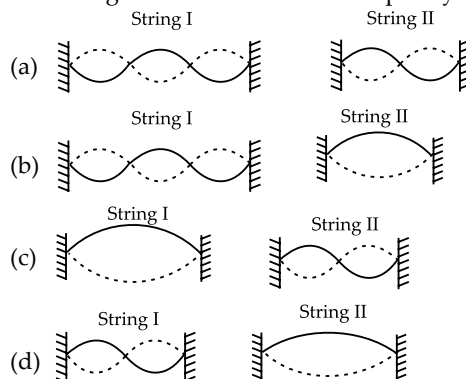


What is the most reasonable explanation of these graphs?

- (a) The first graph shows what happens when the capacitor has a less than E potential initially

and the second shows what happens when it has a greater than E potential initially.

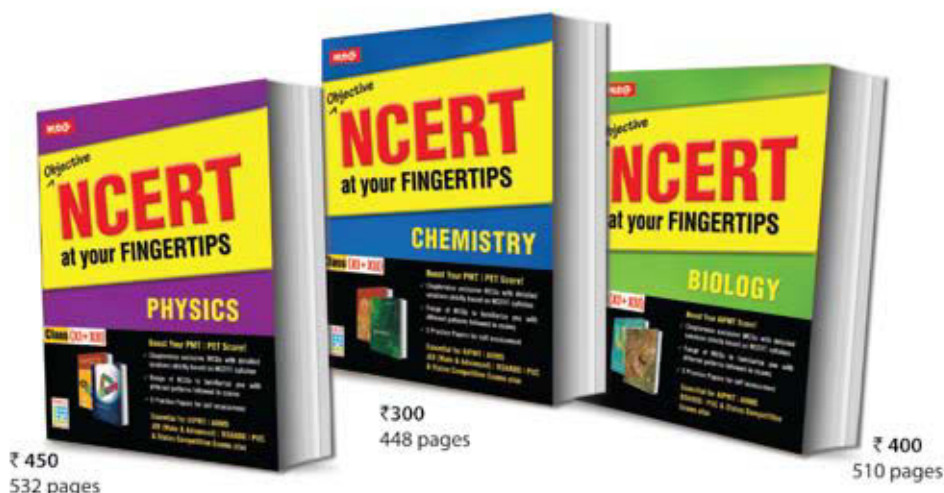
- (b) The first graph shows what happens when the capacitor has a greater than E potential initially and the second shows what happens when it has a less than E potential initially.
(c) The first graph is the correct qualitative shape for any initial potential, but the second is not possible.
(d) The second graph is the correct qualitative shape for any initial potential, but the first is not possible.
10. String I and II have identical lengths and linear mass densities, but string I is under greater tension than string II. The accompanying figure shows four different situations, in which standing wave patterns exist on the two strings. In which situation is it possible that strings I and II oscillating at the same resonant frequency?



100 DAYS OF MARS ORBITER SPACECRAFT

- Mars orbiter Spacecraft, India's first interplanetary probe, was launched by PSLV-C 25 at 1438 hours on November 5, 2013 from Satish Dhawan Space Centre, Sriharikota. In its voyage towards Mars, the mission successfully completes 100 days in space (February 12, 2014).
- Subsequent to six orbit raising manoeuvres around the Earth following the launch, the Trans Mars Injection (TMI) Manoeuvre on December 01, 2013 gave necessary thrust to the spacecraft to escape from Earth and to initiate the journey towards Mars, in a helio-centric Orbit. This journey, of course, is long wherein the spacecraft has to travel 680 million km out of which a travel of 190 million km is completed so far.
- The First Trajectory Correction Manoeuvre (TCM) was conducted on December 11, 2013. The trajectory of the spacecraft, till today, is as expected. Three more TCM operations are planned around April 2014, August 2014 and September 2014.
- The spacecraft health is normal. The spacecraft is continuously monitored by the ground station of ISRO Telemetry, Tracking and Command Network (ISTRAC), located at Bialalu, near Bangalore. Except for a 40 minute break in the Telemetry data received from the spacecraft to the ground station, data has been continuously available for all the 100 days.
- The propulsion system of the spacecraft is configured for TCMs and the Mars Orbit Insertion (MOI) Operation. On February 6, 2014, all the five payloads on Mars Orbiter spacecraft were switched 'ON' to check their health.
- The health parameters of all the payloads the normal. Presently, the spacecraft is at a radio distance of 16 million km causing a one way communication delay of approximately 55 seconds. After travelling the remaining distance of about 490 million km over next 210 days, the spacecraft would be inserted into the Martian Orbit on September 24, 2014.

How to select the correct answer faster?



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Swadeep Biswas says

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PRACTICE PAPER 2Q 14

JEE Advanced

Exam on
25th May

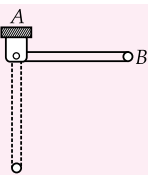
PAPER-1

SECTION - 1

Only One Option Correct Type

This section contains 10 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which ONLY ONE is correct.

- The input resistance of a silicon transistor is $100\ \Omega$. Base current is changed by $40\ \mu\text{A}$ which results in a change in collector current by $2\ \text{mA}$. This transistor is used as a common emitter amplifier with a load resistance of $4\ \text{k}\Omega$. The voltage gain of the amplifier is
(a) 2000 (b) 3000 (c) 4000 (d) 1000
- A photon collides with a stationary hydrogen atom in ground state inelastically. Energy of the colliding photon is $10.2\ \text{eV}$. After a time interval of the order of microsecond, another photon collides with same hydrogen atom inelastically with an energy of $15\ \text{eV}$. What will be observed by the detector?
(a) One photon of energy $10.2\ \text{eV}$ and an electron of energy $1.4\ \text{eV}$.
(b) Two photons of energy $1.4\ \text{eV}$.
(c) Two photons of energy $10.2\ \text{eV}$.
(d) One photon of energy $10.2\ \text{eV}$ and another photon of $1.4\ \text{eV}$.
- A block of mass $1\ \text{kg}$ is attached to one end of spring of force constant $k = 20\ \text{N m}^{-1}$. The other end of the spring is attached to a fixed rigid support. This spring block system is made to oscillate on a rough horizontal surface ($\mu = 0.04$). The initial displacement of the block from the equilibrium position is $a = 30\ \text{cm}$. How many times the block passes from the mean position before coming to rest? (Take $g = 10\ \text{m s}^{-2}$)
(a) 11 (b) 7 (c) 6 (d) 15
- One end of a uniform rod of length l and mass m is hinged at A. It is released from rest from horizontal position AB as shown in figure. The force exerted by the rod on the hinge when it becomes vertical is

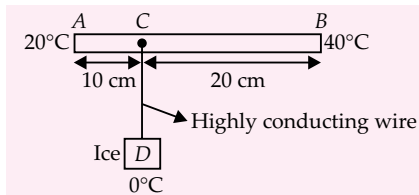


- (a) $\frac{3}{2} mg$ (b) $\frac{5}{2} mg$
(c) $3 mg$ (d) $5 mg$

- An open pipe is suddenly closed at one end with the result that the frequency of third harmonic of the closed pipe is found to be higher by $100\ \text{Hz}$ than the fundamental frequency of the open pipe. The fundamental frequency of the open pipe is
(a) $200\ \text{Hz}$ (b) $300\ \text{Hz}$ (c) $240\ \text{Hz}$ (d) $480\ \text{Hz}$
- A system consists of a uniform charged sphere of radius R and a surrounding medium filled by a charge with the volume density $\rho = \frac{\alpha}{r}$, where α is a positive constant and r is the distance from the centre of the charge. The charge of the sphere for which the electric field intensity E outside the sphere is independent of r is
(a) $\pi R^2 \alpha$ (b) $4\pi R^2 \alpha$
(c) $2\pi R^2 \alpha$ (d) $3\pi R^2 \alpha / 4$
- In a potentiometer experiment, when three cells A, B and C are connected in series the balancing length is found to be $740\ \text{cm}$. If A and B are connected in series balancing length is $440\ \text{cm}$ and for B and C connected in series that is $540\ \text{cm}$. Then the emf of ϵ_A , ϵ_B and ϵ_C are respectively (in volts)
(a) 1, 1.2 and 1.5 (b) 1, 2 and 3
(c) 1.5, 2 and 3 (d) 1.5, 2.5 and 3.5
- A flat glass slab of thickness $6\ \text{cm}$ and refractive index 1.5 is placed in front of a plane mirror. An observer is standing behind the glass slab and looking at the mirror. The actual distance of the observer from the mirror is $50\ \text{cm}$. The distance of his image from himself, as seen by the observer is
(a) $94\ \text{cm}$ (b) $96\ \text{cm}$ (c) $98\ \text{cm}$ (d) $100\ \text{cm}$
- In the figure shown, AB is a rod of length $30\ \text{cm}$ and area of cross-section $1\ \text{cm}^2$ and thermal conductivity $336\ \text{SI units}$. The ends A and B are maintained at temperatures 20°C and 40°C

respectively. A point C of this rod is connected to a box D, containing ice at 0°C , through a highly conducting wire of negligible heat capacity. The rate at which ice melts in the box is

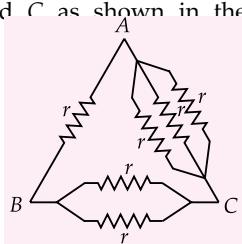
[Assume latent heat of fusion for ice, $L_f = 80 \text{ cal g}^{-1}$]



- (a) 84 mg s^{-1} (b) 84 g s^{-1}
(c) 20 mg s^{-1} (d) 40 mg s^{-1}

10. Six resistances each of value $r = 6 \Omega$ are connected between points A, B and C as shown in the figure.

If R_1 , R_2 and R_3 are the net resistances between A and B, between B and C and between A and C respectively, then $R_1 : R_2 : R_3$ will be equal to



- (a) $6 : 3 : 2$ (b) $1 : 2 : 3$
(c) $5 : 4 : 3$ (d) $4 : 3 : 2$

SECTION - 2

One or More Options Correct Type

This section contains 5 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which ONE or MORE are correct.

11. Two springs A and B have force constants k_1 and k_2 respectively. The ratio of the work done on A to that done on B in increasing their lengths by the same amount is α and the ratio of the work done on A to that done on B when they are stretched with the same force is β . Then

- (a) $\alpha = \frac{k_1}{k_2}$ (b) $\alpha = \frac{k_2}{k_1}$
(c) $\beta = \frac{k_1}{k_2}$ (d) $\beta = \frac{k_2}{k_1}$

12. In a region of space, the electric field is in the X-direction and proportional to x , i.e.; $\vec{E} = E_0 x \hat{i}$. Consider an imaginary cubical volume of edge a , with its edges parallel to the axes of coordinates. The charge inside this volume is

- (a) zero (b) $\epsilon_0 E_0 a^3$
(c) $\frac{1}{\epsilon_0} E_0 a^3$ (d) $\frac{1}{6} \epsilon_0 E_0 a^2$

13. Two long, thin, parallel conductors are kept very close to each other, without touching. One carries a

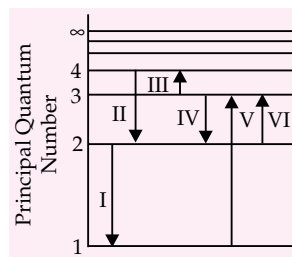
current I , and the other has charge λ per unit length. An electron moving parallel to the conductors is undeflected. Let c is the velocity of light and v is the velocity of electron, then

(a) $v = \frac{\lambda c^2}{I}$ (b) $v = \frac{I}{\lambda}$

(c) $c = \frac{I}{\lambda}$

- (d) The electron may be at any distance from the conductor.

14. The figure shows an energy level diagram for the hydrogen atom. Several transitions are marked as I, II, III, The diagram is only indicative and not to scale. Then,



- (a) the transition in which a Balmer series photon absorbed is VI.
(b) the wavelength of the radiation involved in transition II is 486 nm.
(c) transition IV will occur when a hydrogen atom is irradiated with radiation of wavelength 103 nm.
(d) transition IV will emit the longest wavelength line in the visible portion of the hydrogen spectrum.

15. Consider the motion of a positive point charge in a region where there are simultaneous uniform electric and magnetic fields $\vec{E} = E_0 \hat{j}$ and $\vec{B} = B_0 \hat{j}$. At time $t = 0$, this charge has velocity \vec{v} in the x - y plane, making an angle θ with the x -axis. Which of the following option(s) is (are) correct for time $t > 0$?

- (a) If $\theta = 0^\circ$, the charge moves in a circular path in the x - z plane.
(b) If $\theta = 0^\circ$, the charge undergoes helical motion with constant pitch along the y -axis.
(c) If $\theta = 10^\circ$, the charge undergoes helical motion with its pitch increasing with time, along the y -axis.
(d) If $\theta = 90^\circ$, the charge undergoes linear but accelerated motion along the y -axis.

SECTION - 3

Integer Value Correct Type

This section contains 5 questions. The answer to each question is a single digit integer, ranging from 0 to 9 (both inclusive).

16. The diameter of a convex lens is d . An eye is placed at a distance $3f$ (f being the focal length of the lens)

to the right of the lens at a distance $d/4$ normally below the optic axis so that the image of an object placed on the optic axis to the left of the lens is not visible for a distance greater than $d/4$. The distance of the object is nf . Find the value of n .

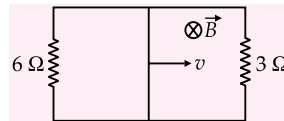
17. If the speed of electron is 35 m s^{-1} with an uncertainty of 5%, the minimum uncertainty in its position is roughly 10^x times the size of the atom, where x is (Take mass of electron = $9.11 \times 10^{-31} \text{ kg}$ and Planck constant = $6.62 \times 10^{-34} \text{ J s}$)
18. A steady current I goes through a wire loop PQR having shape of a right angle triangle with $PQ = 3x$, $PR = 4x$ and $QR = 5x$. If the magnitude of the magnetic field at P due to this loop is

$$k \left(\frac{\mu_0 I}{48\pi x} \right), \text{ find the value of } k.$$

19. A rectangular loop a sliding connector of length $l = 1.0 \text{ m}$ is situated in a uniform magnetic field $B = 2 \text{ T}$ perpendicular to the plane of loop.

Resistance of connector is $r = 2 \Omega$. Two resistances of 6Ω and 3Ω are connected as shown in figure. The external force required to keep the connector moving with a constant velocity $v = 2 \text{ m s}^{-1}$ is

20. A circular platform is mounted on a vertical frictionless axle. Its radius is $r = 2 \text{ m}$ and its moment of inertia is $I = 200 \text{ kg m}^2$. It is initially at rest. A 70 kg man stands on the edge of the platform and begins to walk along the edge at speed $v_0 = 1.0 \text{ m s}^{-1}$ relative to the ground. The angular velocity of the platform is $x \times 10^{-1} \text{ rad s}^{-1}$. The value of x is



PAPER-2

SECTION - 1

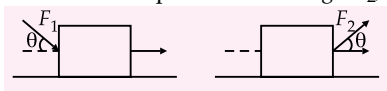
One or More Options Correct Type

This section contains 8 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which ONE or MORE are correct.

1. Two particles are projected from a horizontal plane with the same initial velocity v_0 at two different angles of projection θ_1 and θ_2 , such that their ranges are the same. The ratio of their maximum heights reached is/are

- (a) $\tan^2 \theta_1$ (b) $\cot^2 \theta_2$
(c) $\sin^2 \theta_1 \cos^2 \theta_2$ (d) $\sin^2 \theta_1 \cos^2 \theta_2$

2. In the two cases shown, the coefficient of kinetic friction between the block and the surface is the same and both the identical blocks are moving with the same uniform speed. If $\sin \theta = mg/4F_2$, then



- (a) $F_1 = F_2$ (b) $F_1 < F_2$
(c) $F_1 > F_2$ (d) $F_1 = 2F_2$

3. For two satellites at distance R and $7R$ above the earth's surface, the ratio of their

- (a) total energies is 4 and potential and kinetic energies is 2
(b) potential energies is 4
(c) kinetic energies is 4
(d) total energies is 4

4. Four rods, A , B , C and D of the same length and material but of different radii r , $r\sqrt{2}$, $r\sqrt{3}$ and $2r$ respectively are held between two rigid walls. The temperature of all rods is increased through the same range. If the rods do not bend, then

- (a) the stress in the rods A , B , C and D are in the ratio $1 : 2 : 3 : 4$
(b) the forces on them exerted by the wall are in the ratio $1 : 2 : 3 : 4$
(c) the energy stored in the rods due to elasticity are in the ratio $1 : 2 : 3 : 4$
(d) the strains produced in the rods are in the ratio $1 : 2 : 3 : 4$

5. An α -particle of mass $6.4 \times 10^{-27} \text{ kg}$ and charge $3.2 \times 10^{-19} \text{ C}$ is situated in a uniform electric field of $1.6 \times 10^5 \text{ V m}^{-1}$. The velocity of the particle at the end of $2 \times 10^{-2} \text{ m}$ path when it starts from rest is

- (a) $2\sqrt{3} \times 10^5 \text{ m s}^{-1}$ (b) $8 \times 10^5 \text{ m s}^{-1}$
(c) $16 \times 10^5 \text{ m s}^{-1}$ (d) $4\sqrt{2} \times 10^5 \text{ m s}^{-1}$

6. A charged particle with velocity $\vec{v} = x\hat{i} + y\hat{j}$ moves in a magnetic field $\vec{B} = y\hat{i} + x\hat{j}$. The magnitude of magnetic force acting on the particle is F . Which one of the following statement(s) is/are correct?

- (a) No force will act on particle, if $x = y$.
(b) $F \propto (x^2 - y^2)$ if $x > y$.
(c) The force will act along z -axis, if $x > y$.
(d) The force will act along y -axis, if $y > x$.

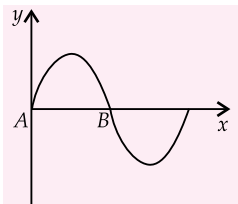
7. A point object is placed at 30 cm from a convex glass lens $\left(\mu_g = \frac{3}{2} \right)$ of focal length 20 cm . The

final image of object will be formed at infinity if

- (a) another concave lens of focal length 60 cm is placed in contact with the convex lens
(b) another convex lens of focal length 60 cm is placed at a distance of 30 cm from the first lens
(c) the whole system is immersed in a liquid of refractive index $4/3$

- (d) the whole system is immersed in a liquid of refractive index $9/8$

8. The tension in a stretched string fixed at both ends is changed by 2%, the fundamental frequency is found to get changed by 15 Hz. Select the correct statement(s)



- (a) Wavelength of the string of fundamental frequency does not change.
 (b) Velocity of propagation of wave changes by 2%.
 (c) Velocity of propagation of wave changes by 1%.
 (d) Original frequency is 1500 Hz.

SECTION - 2

Paragraph Type

This section contains 4 paragraphs each describing theory, experiment, data etc. Eight questions related to four paragraphs with two questions on each paragraph. Each question of a paragraph has only one correct answer among the four choices (a), (b), (c) and (d).

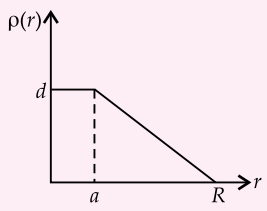
Paragraph for Questions 9 and 10

Two discs A and B are mounted coaxially on a vertical axle. The discs have moments of inertia I and $2I$, respectively about the common axis. Disc A is imparted an initial angular velocity 2ω using the entire potential energy of a spring compressed by a distance x_1 . Disc B is imparted an angular velocity ω by a spring having the same spring constant and compressed by a distance x_2 . Both discs rotate in clockwise direction.

9. The loss of kinetic energy during the process is
 (a) $I\omega^2/2$ (b) $I\omega^2/3$ (c) $I\omega^2/4$ (d) $I\omega^2/6$
10. When disc B is brought in contact with disc A, they acquire a common angular velocity in time t . The average frictional torque during this period is
 (a) $2I\omega/3t$ (b) $9I\omega/2t$ (c) $9I\omega/4t$ (d) $3I\omega/2t$

Paragraph for Questions 11 and 12

The nuclear charge (Ze) is non-uniformly distributed within a nucleus of radius R . The charge density $\rho(r)$ (charge per unit volume) is dependent only on the radial distance r from the centre of the nucleus as shown in figure.



The electric field is only along the radial direction.

11. The electric field at $r = R$ is
 (a) independent of a

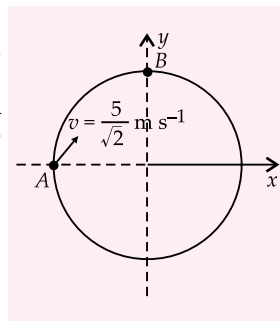
- (b) directly proportional to a
 (c) directly proportional to a^2
 (d) inversely proportional to a

12. For $a = 0$, the value of d (maximum value of ρ as shown in the figure) is

- (a) $\frac{3Ze}{4\pi R^3}$ (b) $\frac{3Ze}{\pi R^3}$
 (c) $\frac{4Ze}{3\pi R^3}$ (d) $\frac{Ze}{3\pi R^3}$

Paragraph for Questions 13 and 14

A ring of mass 200 g and radius 10 m is placed on a smooth horizontal surface with centre at the origin. A small particle of the same mass as ring, is given velocity $\frac{5}{\sqrt{2}} \text{ m s}^{-1}$ from point A (very close to inner surface of the ring) towards point B (at $t = 0$).



Initially particle was not in contact with ring. Assume all collisions between the ring and the particle as perfectly elastic.

13. Particle will collide with point A for the first time after a time interval of
 (a) 8 s (b) 16 s
 (c) 12 s (d) 24 s
14. Co-ordinate of centre of mass of ring when particle reach back to point A for the first time
 (a) (10, 10) (b) (0, 0)
 (c) (20, 20) (d) (-20, -20)

Paragraph for Questions 15 and 16

The key feature of Bohr's theory of spectrum of hydrogen atom is the quantization of angular momentum when an electron is revolving around a proton. We will extend this to a general rotational motion to find quantized rotational energy of a diatomic molecule assuming it to be rigid. The rule to be applied is Bohr's quantization condition.

15. A diatomic molecule has moment of inertia I . By Bohr's quantization condition, its rotational energy in the n^{th} level ($n = 0$ is not allowed) is

- (a) $\frac{1}{n^2} \left(\frac{h^2}{8\pi^2 I} \right)$ (b) $\frac{1}{n} \left(\frac{h^2}{8\pi^2 I} \right)$
 (c) $n \left(\frac{h^2}{8\pi^2 I} \right)$ (d) $n^2 \left(\frac{h^2}{8\pi^2 I} \right)$

16. It is found that the excitation frequency from ground to the first excited state of rotation for the CO molecule is close to $(4/\pi) \times 10^{11}$ Hz. The moment of inertia of CO molecule about its centre of mass is close to

(Take $h = 2\pi \times 10^{-34}$ J s)

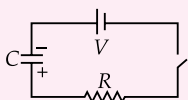
- (a) 2.76×10^{-46} kg m² (b) 1.87×10^{-46} kg m²
(c) 4.67×10^{-47} kg m² (d) 1.17×10^{-47} kg m²

SECTION - 3

Matching List Type

This section contains 4 multiple choice questions. Each question has matching lists. The codes for the lists have choices (a), (b), (c) and (d) out of which ONLY ONE is correct.

17. In the shown circuit initially capacitor has some charge, the switch is closed at $t = 0$.



List - I Variables

(P) Charge v/s time

(Q) Current v/s time

(R) Charge v/s current

(S) Energy stored in capacitor v/s time

- | P | Q | R | S |
|-------|---|---|---|
| (a) 1 | 2 | 3 | 4 |
| (b) 3 | 1 | 2 | 4 |
| (c) 1 | 4 | 2 | 3 |
| (d) 3 | 4 | 1 | 2 |

List - II Possible graphs

(1)

(2)

(3)

(4) None of these

18. In the following, List-I lists some physical quantities and the List-II gives approximate energy values associated with some of them. Choose the appropriate value of energy from List-II for each of the physical quantities in List-I.

List-I

(P) Energy of thermal neutrons

(Q) Energy of X-rays

(R) Binding energy per nucleon

(S) Photoelectric threshold of a metal

List-II

(1) 0.025 eV

(2) 8 MeV

(3) 3 eV

(4) 10 keV

- | | P | Q | R | S |
|-----|---|---|---|---|
| (a) | 1 | 2 | 3 | 4 |
| (b) | 3 | 1 | 2 | 4 |
| (c) | 1 | 4 | 2 | 3 |
| (d) | 3 | 4 | 1 | 2 |

19. List-I gives some devices and List-II gives some processes on which the functioning of these devices depend. Match the device in List-I with the processes in List-II.

List - I

(P) Bimetallic strip

(Q) Steam engine

(R) Incandescent lamp

(S) Electric fuse

List - II

(1) Radiation from a hot body

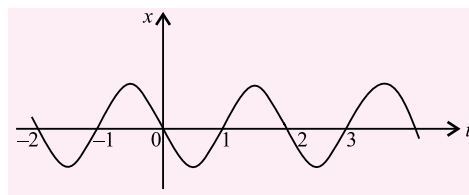
(2) Energy conversion

(3) Melting

(4) Thermal expansion of solids

- | P | Q | R | S |
|-------|---|---|---|
| (a) 1 | 2 | 3 | 4 |
| (b) 4 | 3 | 2 | 4 |
| (c) 3 | 4 | 2 | 1 |
| (d) 4 | 2 | 1 | 3 |

20. Figure gives the x - t plots of a particle executing one-dimensional simple harmonic motion.



Match the List-I with List-II.

List - I Time

(P) At $t = -1.2$ s

(Q) At $t = -0.3$ s

(R) At $t = 0.3$ s

(S) At $t = 1.2$ s

List - II

Signs of position (x), velocity (v) and acceleration (a)

(1) $x < 0, v < 0, a > 0$

(2) $x > 0, v > 0, a < 0$

(3) $x > 0, v < 0, a < 0$

(4) $x < 0, v > 0, a > 0$

- | P | Q | R | S |
|-------|---|---|---|
| (a) 4 | 3 | 1 | 2 |
| (b) 3 | 1 | 2 | 4 |
| (c) 4 | 3 | 2 | 1 |
| (d) 3 | 4 | 1 | 2 |

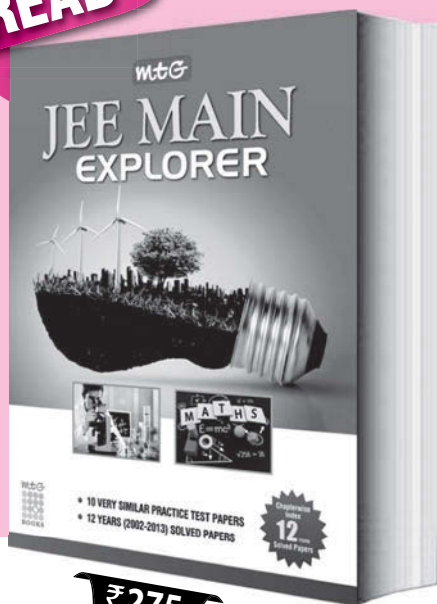
SOLUTIONS

Paper 1

1. (a): Here, collector current, $I_{\text{out}} = 2 \text{ mA} = 2 \times 10^{-3} \text{ A}$
Base current, $I_{\text{in}} = 40 \mu\text{A} = 40 \times 10^{-6} \text{ A}$
Input resistance $R_{\text{in}} = 100 \Omega$
Output resistance $R_{\text{out}} = 4 \text{ k}\Omega = 4 \times 10^3 \Omega$

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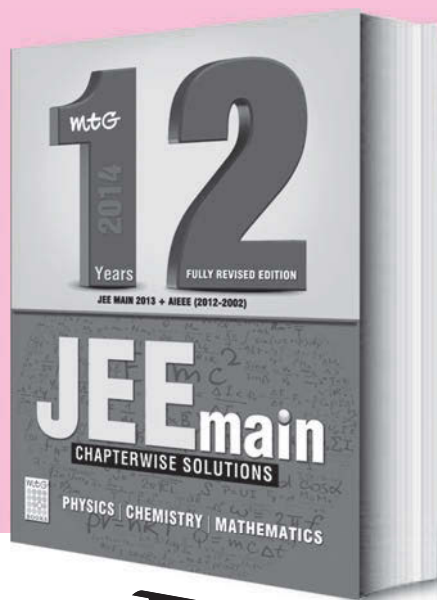
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$$\begin{aligned}\text{Voltage gain} &= \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{I_{\text{out}}}{I_{\text{in}}} \times \frac{R_{\text{out}}}{R_{\text{in}}} \\ &= \frac{2 \times 10^{-3} \text{ A}}{40 \times 10^{-6} \text{ A}} \times \frac{4 \times 10^{-3} \Omega}{100 \Omega} \\ &= 2000\end{aligned}$$

2. (a): The photon of energy 10.2 eV excites the electron from $n = 1$ to $n = 2$ as
 $E_2 - E_1 = -3.4 \text{ eV} - (-13.6 \text{ eV}) = 10.2 \text{ eV}$
 The electron returns to the ground state in less than a microsecond and releases a photon of energy 10.2 eV. As the ionisation energy is 13.6 eV, the second photon of 15 eV energy ionises the atom by ejecting an electron and the balance of energy (15 eV - 13.6 eV = 1.4 eV) is retained by the ejected electron.

3. (b): Let the initial amplitude decreases to a_1 to the other side i.e., after the first sweep, decrease in elastic potential energy
 = work done against friction

$$\text{or } \frac{1}{2}ka^2 - \frac{1}{2}ka_1^2 = \mu mg(a + a_1)$$

$$\text{or } \frac{1}{2}k(a + a_1)(a - a_1) = \mu mg(a + a_1)$$

$$\text{or } a - a_1 = \frac{2\mu mg}{k} \quad \dots(i)$$

$$\text{Similarly, } a_1 - a_2 = \frac{2\mu mg}{k} \quad \dots(ii)$$

$$\dots\dots\dots$$

$$a_{n-1} - a_n = \frac{2\mu mg}{k} \quad \dots(iii)$$

Adding all the above equations

$$a - a_n = \frac{2n\mu mg}{k} \quad \dots(iv)$$

The block stops when,

$$\mu mg = ka_n \quad \text{or} \quad a_n = \frac{\mu mg}{k}$$

Substituting in equation (iv) we get,

$$(2n + 1) \left(\frac{\mu mg}{k} \right) = a$$

$$\text{or } (2n + 1) = \frac{ka}{\mu mg} = \frac{20 \times 0.3}{0.04 \times 1 \times 10} = 15$$

$$\text{or } 2n = 15 - 1$$

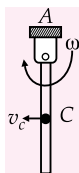
$$\therefore n = 7$$

4. (b): The rod will rotate about A. Therefore, from conservation of mechanical energy, Decrease in gravitational potential energy = increase in rotational kinetic energy about A

$$\text{or } mg \frac{l}{2} = \frac{1}{2} I_A \omega^2$$

$$\text{or } mg \frac{l}{2} = \frac{1}{2} \left(\frac{ml^2}{3} \right) \omega^2$$

$$\therefore \omega^2 = \frac{3g}{l} \quad \dots(i)$$



Centripetal force of COM of rod in this position is

$$m \frac{l}{2} \omega^2 = \frac{3mg}{2} \quad (\text{towards A})$$

Let F be the force exerted by the hinge on the rod upwards. Then

$$F - mg = \frac{3mg}{2} \quad \therefore F = \frac{5}{2}mg$$

or force exerted by the rod on the hinge is $\frac{5}{2}mg$ downwards.

5. (a): Length of organ pipe is same in both the cases. Fundamental frequency of open pipe is

$v_1 = \frac{v}{2l}$ and frequency of third harmonic of closed pipe will be

$$v_2 = 3 \left(\frac{v}{4l} \right)$$

Given that, $v_2 = v_1 + 100$

$$\text{or } v_2 - v_1 = 100$$

$$\text{or } 3 \left(\frac{v}{4l} \right) - \left(\frac{1}{2} \right) \left(\frac{v}{l} \right) = 100$$

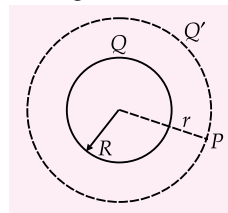
$$\Rightarrow \frac{v}{4l} = 100 \text{ Hz}$$

$$\therefore \frac{v}{2l} \text{ or } v_1 = 200 \text{ Hz}$$

Therefore, fundamental frequency of the open pipe is 200 Hz.

6. (c): $\oint E_p dS = \frac{Q + Q'}{\epsilon_0} \quad \dots(i)$

where Q' is the charge outside the sphere.



$$\begin{aligned}Q' &= \int_R^r dV = \int_R^r \frac{\alpha}{r} \times 4\pi r^2 dr \\ &= 4\pi \alpha \left(\frac{r^2}{2} \right)_R^r = 4\pi \alpha \left(\frac{r^2}{2} - \frac{R^2}{2} \right) = 2\pi \alpha (r^2 - R^2)\end{aligned}$$

From equation (i)

$$E_p \times 4\pi r^2 = \frac{Q + 2\pi \alpha (r^2 - R^2)}{\epsilon_0}$$

$$E_p = \frac{Q}{4\pi r^2 \epsilon_0} + \frac{\alpha}{2\epsilon_0} - \frac{\alpha R^2}{2r^2 \epsilon_0}$$

$$E \text{ is independent of } r \text{ if } \frac{Q}{4\pi r^2 \epsilon_0} - \frac{\alpha R^2}{2r^2 \epsilon_0} = 0$$

$$Q = 2\pi R^2 \alpha$$

7. (a): Let ε_A , ε_B and ε_C be the emf of three cells A, B and C respectively.

As per question

$$\varepsilon_A + \varepsilon_B + \varepsilon_C = kl_1 = k \times 740 \quad \dots(i)$$

$$\varepsilon_A + \varepsilon_B = kl_2 = k \times 440 \quad \dots(ii)$$

$$\varepsilon_B + \varepsilon_C = kl_3 = k \times 540 \quad \dots(iii)$$

Inserting the value of $\varepsilon_A + \varepsilon_B$ from (ii) into (i), we get

$$\varepsilon_C = 300k$$

Inserting the value of $\varepsilon_B + \varepsilon_C$ from (iii) into (i), we get

$$\varepsilon_A = 200k$$

Inserting this value of ε_A into (ii), we get

$$\varepsilon_B = 240k$$

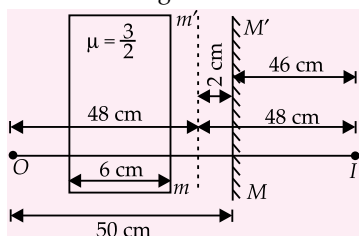
$$\therefore \varepsilon_A : \varepsilon_B : \varepsilon_C = 200k : 240k : 300k \\ = 10 : 12 : 15 = 1 : 1.2 : 1.5$$

$$\Rightarrow \varepsilon_A = 1 \text{ V}, \varepsilon_B = 1.2 \text{ V}, \varepsilon_C = 1.5 \text{ V}$$

8. (b): As the thickness of glass slab is 6 cm and its refractive index $\mu = \frac{3}{2}$, so shift produced by it will be

$$x = t \left(1 - \frac{1}{\mu} \right) = 6 \left[1 - \frac{2}{3} \right] = 2 \text{ cm}$$

So the glass slab will shift the mirror from MM' to mm' as shown in the figure.



The distance of object from this virtual mirror will be

$$= 50 - x = 50 - 2 = 48 \text{ cm}$$

This virtual mirror will form the image of object O at a distance 48 cm behind it and so the distance of image from actual mirror MM' will be

$$= 48 - 2 = 46 \text{ cm}$$

[As mm' is 2 cm in front of MM']

So the distance of image as seen by the observer is $= 50 \text{ cm} + 46 \text{ cm} = 96 \text{ cm}$

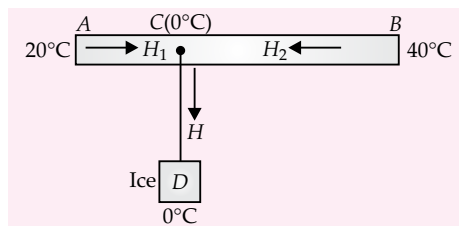
9. (d): Thermal resistance of $AC = \frac{L}{KA}$

$$= \frac{0.1}{336 \times 1 \times 10^{-4}} = \frac{10^3}{336} = R (\text{suppose})$$

$$\text{Thermal resistance of } BC = \frac{0.2}{336 \times 10^{-4}} = 2R$$

Temperature of $C = 0^\circ\text{C}$

$$\therefore H_1 = \frac{20}{R}; H_2 = \frac{40}{2R} = \frac{20}{R}$$



$$\therefore H = H_1 + H_2 = \frac{40}{R} = \frac{40 \times 336}{10^3} \\ = \frac{13440}{10^3} = 13.44 \text{ W}$$

Rate of melting of ice

$$= \frac{H}{L_f} = \frac{13.44}{4.2 \times 80} \text{ g s}^{-1} = 40 \text{ mg s}^{-1}$$

10. (c)

11. (a,d): As, $F_1 = k_1x$, $F_2 = k_2x$.

$$\text{Work done } W_1 = \frac{1}{2}k_1x^2 \text{ and } W_2 = \frac{1}{2}k_2x^2$$

$$\text{or } \alpha = \frac{W_1}{W_2} = \frac{k_1}{k_2}$$

When the springs are stretched by the same force F , the extensions in springs A and B are x_1 and x_2 respectively which are given by

$$F = k_1x_1 = k_2x_2 \text{ or } \frac{x_1}{x_2} = \frac{k_2}{k_1} \quad \dots(i)$$

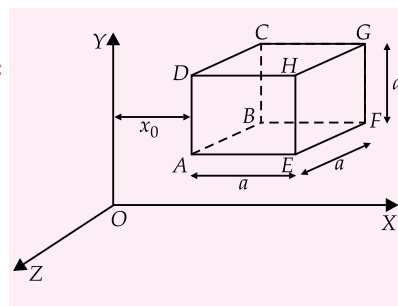
$$\text{Work done } W'_1 = \frac{1}{2}k_1x_1^2 \text{ and } W'_2 = \frac{1}{2}k_2x_2^2$$

$$\therefore \frac{W'_1}{W'_2} = \frac{k_1}{k_2} \cdot \frac{x_1^2}{x_2^2} \quad \dots(ii)$$

Using (i) and (ii) we get

$$\beta = \frac{W'_1}{W'_2} = \frac{k_1}{k_2} \cdot \frac{k_2^2}{k_1^2} = \frac{k_2}{k_1}$$

12. (b):



The field at the face $ABCD = E_0x_0 \hat{i}$

$$\therefore \text{flux over the face } ABCD = -(E_0x_0)a^2.$$

The negative sign arises as the field is directed into the cube.

The field at the face $EFGH = E_0(x_0 + a) \hat{i}$.

\therefore flux over the face $EFGH = E_0(x_0 + a)a^2$.

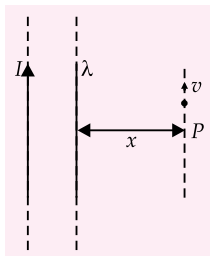
The flux over the other four faces is zero as the field is parallel to the surfaces.

\therefore total flux over the cube $= E_0 a^3 = \frac{1}{\epsilon_0} q$,

where q is the total charge inside the cube.

$\therefore q = \epsilon_0 E_0 a^3$.

13. (a,d) :



At P , electric field, $E = \frac{\lambda}{2\pi\epsilon_0 x}$ (to the right),

and magnetic field, $B = \frac{\mu_0 I}{2\pi x}$ (into the paper)

For no deflection, $E = vB$ or $v = \frac{E}{B}$

$$\text{or } v = \frac{\lambda}{2\pi\epsilon_0 x} \times \frac{2\pi x}{\mu_0 I} = \frac{\lambda}{I \epsilon_0 \mu_0} = \frac{\lambda c^2}{I}$$

$$\left(\because c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \right)$$

14. (a,b,d): For Balmer series, $n_1 = 2$, $n_2 = 3, 4, \dots$ (lower) (higher)

\therefore In transition (VI), photon of Balmer series is absorbed.

In transition II

$$E_2 = -3.4 \text{ eV}, E_4 = -0.85 \text{ eV}$$

$$\Delta E = 2.55 \text{ eV}$$

$$\Delta E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{\Delta E}$$

$$\lambda = 486 \text{ nm}$$

$$\text{Wavelength of radiation} = 103 \text{ nm} = 1030 \text{ \AA}$$

$$\therefore \Delta E = \frac{12400 \text{ eV \AA}}{1030 \text{ \AA}}$$

$$\approx 12.0 \text{ eV}$$

So difference of energy should be 12.0 eV (approx)

Hence $n_1 = 1$ and $n_2 = 3$

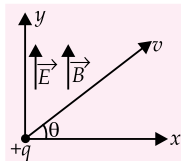
$$(-13.6 \text{ eV}) - (-1.51 \text{ eV})$$

\therefore Transition is V.

For longest wavelength, energy difference should be minimum.

So in visible portion of hydrogen atom, minimum energy emitted is in transition IV.

15. (c, d) :



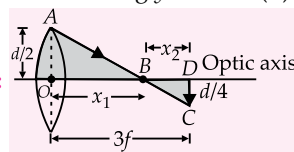
Here, $\vec{E} = E_0 \hat{j}$, $\vec{B} = B_0 \hat{j}$

If $\theta = 0^\circ$, then due to magnetic force path is circular but due to electric force qE_0 (\uparrow) q will have accelerated motion along y -axis. So combined path of q will be a helical path with variable pitch. So (a) and (b) are wrong.

If $\theta = 10^\circ$ then due to $v \cos \theta$, path is circular and due to qE_0 and $v \sin \theta$, q has accelerated motion along y -axis so combined path is a helical path with variable pitch. So (c) is correct.

If $\theta = 90^\circ$ then $F_B = 0$ and due to qE_0 motion is accelerated along y -axis. So (d) is correct.

16. (2) :



From similar triangles AOB and BDC

$$\frac{OB}{BD} = \frac{AO}{CD} \text{ or } \frac{x_1}{x_2} = \frac{(d/2)}{(d/4)}$$

$$\text{or } x_1 = 2x_2$$

$$\text{As } x_1 + x_2 = 3f, 2x_2 + x_2 = 3f \text{ or } x_2 = f$$

$$\text{i.e. } x_1 = 2f$$

$$\therefore n = 2.$$

17 (5): Uncertainty in electron's momentum,

$$\Delta p_x = (0.05)mv$$

$$= 0.05(9.11 \times 10^{-31} \text{ kg})(35 \text{ m s}^{-1})$$

$$= 1.59 \times 10^{-31} \text{ kg m s}^{-1}$$

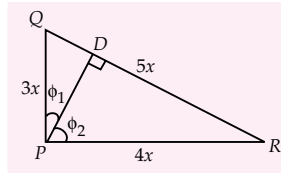
Minimum uncertainty in electron's position,

$$\Delta x = \frac{h}{2\pi \Delta p_x}$$

$$= \frac{6.62 \times 10^{-34} \text{ J s}}{2\pi (1.59 \times 10^{-30} \text{ kg m s}^{-1})}$$

$$= 6.64 \times 10^{-5} \text{ m} \approx 10^5 \text{ times the size of atom}$$

18. (7) :



Using the concept of area of triangle

$$\frac{1}{2} \times PD \times 5x = \frac{1}{2} \times 3x \times 4x$$

$$\therefore PD = \frac{12x}{5}$$

$$QD = \sqrt{(PQ)^2 - (PD)^2} = \sqrt{9x^2 - \frac{144x^2}{25}} = \frac{9x}{5}$$

$$\text{and } DR = 5x - \frac{9x}{5} = \frac{16x}{5}$$

Magnetic field at P due to current elements PQ and PR is zero as the point P is on the conductor. Therefore, magnetic field at P due to current element QR is

$$B = \frac{\mu_0 I}{4\pi PD} (\sin \phi_1 + \sin \phi_2)$$

$$B = \frac{\mu_0 I \times 5}{4\pi \times 12x} \left(\frac{(9x/5)}{3x} + \frac{(16x/5)}{4x} \right)$$

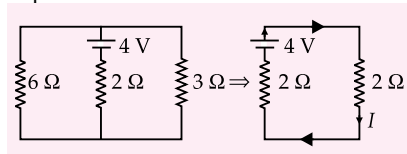
$$= \frac{\mu_0 I 5}{48\pi x} \left(\frac{3}{5} + \frac{4}{5} \right) = \frac{7\mu_0 I}{48\pi x} \quad \therefore k = 7$$

19. (2): Motional emf $e = Bvl$

$$e = 2 \times 2 \times 1 = 4 \text{ V}$$

This acts as a cell of emf $\varepsilon = 4 \text{ V}$ and internal resistance $r = 2 \Omega$.

The simple circuit can be drawn as follows



\therefore Current through the connector

$$I = \frac{4}{2+2} = 1 \text{ A}$$

Magnetic force on connector $F_m = IlB$

$$= (1) \times (1) \times (2) = 2 \text{ N} \quad (\text{towards left})$$

Therefore, to keep the connector moving with a constant velocity a force of 2 N will have to be applied towards right.

- 20 (7): Net external torque is zero. Therefore, angular momentum of system will remain conserved, i.e.; $L_i = L_f$

Initial angular momentum $L_i = 0$

\therefore final angular momentum should also be zero.

or angular momentum of man

= angular momentum of platform in opposite direction

$$\text{or } mv_0 r = I\omega$$

$$\text{or } \omega = \frac{mv_0 r}{I} = \frac{70 \times 1 \times 2}{200}$$

$$\therefore \omega = 0.7 \text{ rad s}^{-1} = 7 \times 10^{-1} \text{ rad s}^{-1}$$

$$\therefore x = 7$$

Paper 2

1. (a, b, c): Here, $\theta_2 = 90^\circ - \theta_1$

The ratio of maximum height reached =

$$\frac{h_1}{h_2} = \frac{\sin^2 \theta_1}{\sin^2 (90^\circ - \theta_1)} = \tan^2 \theta_1 = \cot^2 \theta_2$$

2. (c, d): If θ is the angle made by the direction of force with the horizontal, we have

$$F_1 \cos \theta = \mu(mg + F_1 \sin \theta) \text{ and}$$

$$F_2 \cos \theta = \mu(mg - F_2 \sin \theta).$$

Clearly $F_1 > F_2$ so that option (c) is correct.

If $\sin \theta = \frac{mg}{4F_2}$, two relations written above becomes

$$F_1 \cos \theta = \mu \left(mg + \frac{mgF_1}{4F_2} \right) \text{ and}$$

$$F_2 \cos \theta = \mu \left(mg - \frac{mgF_2}{4F_2} \right).$$

$$\text{Thus, } \frac{F_1}{F_2} = \frac{1 + (F_1/4F_2)}{(3/4)}.$$

$$\Rightarrow F_1 = 2F_2$$

3. (b, c, d)

4. (b, c): Thermal force $= Y A \alpha d\theta = Y \pi r^2 \alpha d\theta$

$$r_1 = r, r_2 = r\sqrt{2}, r_3 = r\sqrt{3}, r_4 = 2r,$$

The ratio of forces on them exerted by the wall,

$$F_1 : F_2 : F_3 : F_4 = 1 : 2 : 3 : 4$$

Thermal stress $= Y \alpha d\theta$

As Y and α are same for all the rods, hence stress developed in each rod will be same.

As strain $= \alpha d\theta$, so strain will also be same.

$$\text{Energy stored} = \frac{1}{2} Y (\text{strain})^2 \times A \times L$$

$$\therefore E_1 : E_2 : E_3 : E_4 = 1 : 2 : 3 : 4$$

5. (d)

6. (a, b, c): Here $\vec{v} = x\hat{i} + y\hat{j}$
 $\vec{B} = y\hat{i} + x\hat{j}$

$$\text{If } x = y \text{ then } \vec{v} \parallel \vec{B} \text{ i.e.; } \vec{F} = 0$$

Hence, option (a) is correct

$$\text{As } \vec{F} = q(\vec{v} \times \vec{B}) = q[(x\hat{i} + y\hat{j}) \times (y\hat{i} + x\hat{j})]$$

$$= (x^2 - y^2)\hat{k}$$

Now, if $x > y$, $F \propto x^2 - y^2$ and force is along z-axis. But if $y > x$, force will be along negative z-axis.

\therefore Option (b) and (c) are also correct.

7. (a, d): Final image is formed at infinity if the combined focal length of the two lenses (in contact) becomes 30 cm

$$\frac{1}{30} = \frac{1}{20} + \frac{1}{f}$$

i.e. when another concave lens of focal length 60 cm is kept in contact with the first lens.

Similarly, let μ be the refractive index of a liquid in which focal length of the given lens becomes 30 cm. Then

$$\frac{1}{20} = \left(\frac{3}{2} - 1\right) \left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \dots(i)$$

$$\frac{1}{30} = \left(\frac{3/2}{\mu} - 1\right) \left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \dots(ii)$$

From equations (i) and (ii), we get

$$\mu = \frac{9}{8}$$

8. (a, c, d): Wavelength depends on length which is fixed. Thus, wavelength does not change.

Further $v = \sqrt{T/m}$ or $v \propto T^{1/2}$

$$\therefore \text{percentage change in } v = \frac{1}{2} \times \text{percentage change in } T$$

$$= \frac{1}{2} (2) = 1\%$$

i.e. Speed and hence frequency will change by 1%.

Change in frequency is 15 Hz which is 1% of 1500 Hz.

Therefore, original frequency should be 1500 Hz.

9. (b): Loss in KE = $K_i - K_f$
- $$= \left[\frac{1}{2} I (2\omega)^2 + \frac{1}{2} (2I) (\omega)^2 \right] - \frac{1}{2} (I + 2I) \left(\frac{4\omega}{3} \right)^2 = \frac{I\omega^2}{3}$$
- [as from conservation of angular momentum,

$$I(2\omega) + (2I)\omega = (I + 2I)\omega_f \text{ where } \omega_f = \frac{4\omega}{3}]$$

10. (a): Considering the disc B (with moment of inertia $2I$ and angular velocity ω),

$$\omega_f = \omega_i + \alpha t, \quad \alpha = \frac{\omega_f - \omega_i}{t} = \frac{(4\omega/3) - \omega}{t} = \frac{\omega}{3t}$$

$$\text{Thus, } \tau = (2I)\alpha = \frac{2I\omega}{3t}$$

11. (a): Electric field at the surface of the nucleus,

$$E = k_e \frac{Q}{R^2} \text{ i.e. } r = R$$

Which is independent of a .

12. (b)

13. (b): Time taken by particle to move from A to B is

$$t_1 = \frac{\sqrt{2}R}{\frac{5}{\sqrt{2}}} = \frac{2R}{5} = \frac{20}{5} = 4 \text{ second.}$$

Since collision is perfectly elastic, relative velocity between ring and particles will not change. Hence after 4th time it reaches to A. i.e. $4 \times 4 = 16$ s

14. (c): For the particle, x component of velocity (2 initial)

$$= \frac{5}{2} \text{ m s}^{-1}$$

$$y \text{ component of velocity (2 initial)} = \frac{5}{2} \text{ m s}^{-1}$$

For 8 second ring move along x direction and for 8 second it move along y direction. So, its centre of mass = $(2R, 2R) = (20, 20)$

15. (d): Angular momentum, $L = \frac{nh}{2\pi}$
Kinetic energy of rotation,

$$K_r = \frac{L^2}{2I} \quad (\because K_{\text{translation}} = \frac{p^2}{2m})$$

$$\text{Thus, } K_r = \frac{(nh/2\pi)^2}{2I} = n^2 \left(\frac{h^2}{8\pi^2 I} \right)$$

16. (b): As, $h\nu = (K_r)_2 - (K_r)_1 = \frac{4h^2}{8\pi^2 I} - \frac{h^2}{8\pi^2 I} = \frac{3h^2}{8\pi^2 I}$

$$I = \frac{3h}{8\pi^2 \nu} = \frac{3(2\pi \times 10^{-34} \text{ J s})}{8\pi^2 \left(\frac{4}{\pi} \times 10^{11} \text{ s}^{-1} \right)}$$

$$= 0.1875 \times 10^{-45} \text{ kg m}^2 = 1.87 \times 10^{-46} \text{ kg m}^2$$

17. (a)

18. (c):

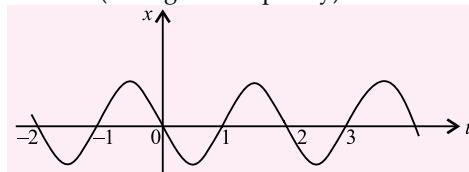
List-I

List-II

(P) Energy of thermal neutrons	(1) 0.025 eV
(Q) Energy of X-rays	(4) 10 keV
(R) Binding energy per nucleon	(2) 8 MeV
(S) Photoelectric threshold of a metal	(3) 3 eV

19. (d)

20. (a): In SHM, acceleration, $a = -\omega^2 x$
where ω is (i.e. angular frequency) constant.



At $t = -1.2$ s, $x < 0$

The slope of $x-t$ is positive, hence v is positive.

Since $a = -\omega^2 x$, hence a is positive.

\therefore At $t = -1.2$ s, $x < 0$, $v > 0$, $a > 0$

P - 4

At $t = -0.3$ s, $x > 0$

The slope of $x-t$ is negative, hence v is negative.

Since $a = -\omega^2 x$, hence $a < 0$

\therefore At $t = -0.3$ s, $x > 0$, $v < 0$, $a < 0$

Q - 3

At $t = 0.3$ s, $x < 0$

The slope of $x-t$ is negative, hence v is negative.

Since $a = -\omega^2 x$, hence $a > 0$

\therefore At $t = 0.3$ s, $x < 0$, $v < 0$, $a > 0$

R - 1

At $t = 1.2$ s, $x > 0$

The slope of $x-t$ is positive, hence v is positive.

Since $a = -\omega^2 x$, hence $a < 0$

\therefore At $t = 1.2$ s, $x > 0$, $v > 0$, $a < 0$

S - 2

JEE

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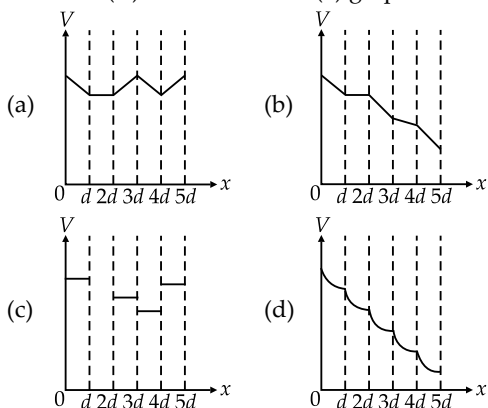
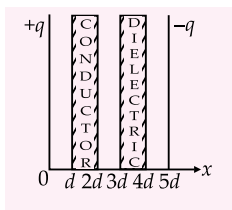
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PRACTICE PAPER 2Q 14

JEE Main

Exam on
6th April

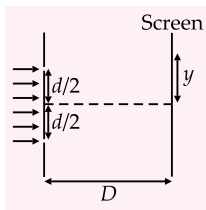
1. The distance between plates of a parallel plate capacitor is $5d$. The positively charged plate is at $x=0$ and negatively charged plate is at $x=5d$. Two slabs one of conductor and the other of a dielectric of same thickness d are inserted between the plates as shown in figure. Potential (V) versus distance (x) graph will be



2. Let K_1 be the maximum kinetic energy of photoelectrons emitted by light of wavelength λ_1 and K_2 corresponding to wavelength λ_2 . If $\lambda_1 = 2\lambda_2$ then
- (a) $2K_1 = K_2$ (b) $K_1 = 2K_2$
(c) $K_1 < K_2/2$ (d) $K_1 > 2K_2$
3. A rigid circular loop of radius r and mass m lies in the x - y plane on a flat table and has a current I flowing in it. At this particular place, the earth's magnetic field is $\vec{B} = B_x \hat{i} + B_z \hat{k}$. What is the value I so that one edge of the loop lifts from the table?

- (a) $\frac{mg}{\pi r \sqrt{B_x^2 + B_z^2}}$ (b) $\frac{mg}{\pi r B_z}$
(c) $\frac{mg}{\pi r B_x}$ (d) $\frac{mg}{\pi r \sqrt{B_x B_z}}$

4. In the Young's double slit experiment apparatus shown in figure, the ratio of maximum to minimum intensity on the screen is 9. The wavelength of light used is λ , then the value of y is



- (a) $\frac{\lambda D}{d}$ (b) $\frac{\lambda D}{2d}$
(c) $\frac{\lambda D}{3d}$ (d) $\frac{\lambda D}{4d}$

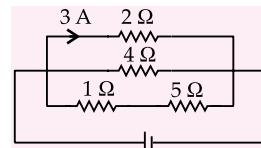
5. The frequency of a sonometer wire is ν , but when the weights producing the tensions are completely immersed in water the frequency becomes $\nu/2$ and on immersing the weights in a certain liquid the frequency becomes $\nu/3$. The specific gravity of the liquid is

- (a) $\frac{4}{3}$ (b) $\frac{16}{9}$ (c) $\frac{15}{12}$ (d) $\frac{32}{27}$

6. Find the phase velocity of electromagnetic wave having electron density and frequency for D layer, $N = 400$ electron per c.c., $\nu = 200$ kHz.

- (a) $3 \times 10^8 \text{ m s}^{-1}$ (b) $3.4 \times 10^8 \text{ m s}^{-1}$
(c) $6.9 \times 10^8 \text{ m s}^{-1}$ (d) $1.1 \times 10^9 \text{ m s}^{-1}$

7. A current of 3 A flows through the 2Ω resistor shown in the circuit below. The power dissipated in the 5Ω resistor is



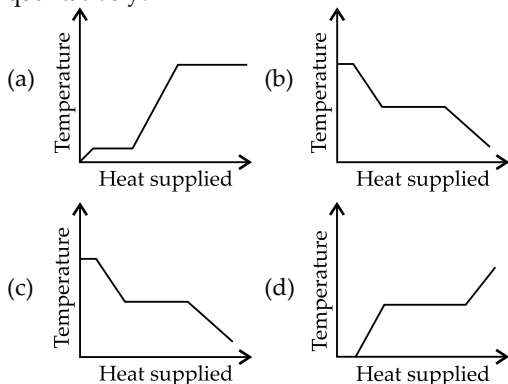
- (a) 1 W
(b) 5 W
(c) 4 W
(d) 2 W

8. Two large parallel copper plates are L m apart and have a uniform electric field between them. An electron is released from the negative plate at the same time, a proton is released from the positive plate. The gravity and force on the particles on each other are to be neglected. The two particles cross each other at a distance r from the positive plate. Then r is

- (a) $L/2$ (b) $\frac{m_p L}{m_e}$
 (c) $\frac{m_e L}{(m_e + m_p)}$ (d) $\frac{m_p L}{(m_e + m_p)}$

9. When a dc voltage of 200 V is applied to a coil of self inductance $2\sqrt{3}/\pi$ H, a current of 1 A flows through it. But by replacing dc source with ac source of 200 V, the current in the coil is reduced to 0.5 A. Then the frequency of ac supply is
 (a) 100 Hz (b) 75 Hz
 (c) 50 Hz (d) 30 Hz

10. A block of ice at -10°C is slowly heated and converted to steam at 100°C . Which of the following curves represents the phenomena qualitatively?

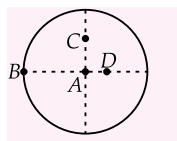


11. Two wires of same dimension but resistivities ρ_1 and ρ_2 are connected in series. The equivalent resistivity of the combination is

- (a) $\rho_1 + \rho_2$ (b) $\frac{1}{2}(\rho_1 + \rho_2)$
 (c) $\sqrt{\rho_1 \rho_2}$ (d) $2(\rho_1 + \rho_2)$

12. The moment of inertia of a uniform circular disc is maximum about an axis perpendicular to the disc and passing through

- (a) B
 (b) C
 (c) D
 (d) A



13. When an ideal monoatomic gas is heated at constant pressure, the fraction of the heat energy supplied which increases the internal energy of the gas is

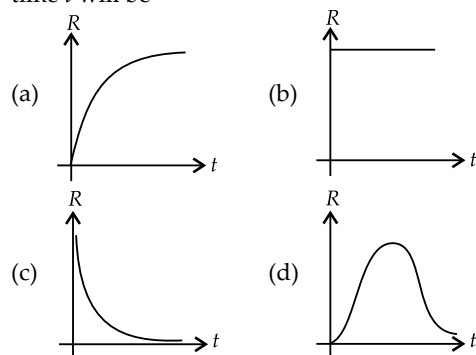
- (a) $2/5$ (b) $3/5$ (c) $3/7$ (d) $5/7$

14. A small current element of length dl is placed at $(1, 1, 0)$ and is carrying current in $+z$ direction. If magnetic field at origin be \vec{B}_1 and at point

$(2, 2, 0)$ be \vec{B}_2 , then

- (a) $\vec{B}_1 = \vec{B}_2$ (b) $|\vec{B}_1| = 2|\vec{B}_2|$
 (c) $\vec{B}_1 = -\vec{B}_2$ (d) $\vec{B}_1 = -2\vec{B}_2$

15. Two beams of light having intensities I and $4I$ interfere to produce a fringe pattern on a screen. The phase difference between the beams is $\pi/2$ at point A and π at point B. Then the difference between the resultant intensities at A and B is
 (a) $2I$ (b) $4I$ (c) $5I$ (d) $7I$
16. A radioactive nucleus X decays to a stable nucleus Y. Then the graph of rate of formation of Y against time t will be



Directions : Question numbers 17-18 contain Statement-1 and Statement-2. Of the four choices given choose the one that best describes the two statements.

- (a) Statement-1 is false, Statement-2 is true.
 (b) Statement-1 is true, Statement-2 is true, Statement-2 is the correct explanation of Statement-1.
 (c) Statement-1 is true, Statement-2 is true, Statement-2 is not the correct explanation of Statement-1.
 (d) Statement-1 is true, Statement-2 is false.

17. **Statement 1 :** The threshold frequency of photoelectric effect supports the particle nature of light.

Statement 2 : If frequency of incident light is less than the threshold frequency, electrons are not emitted from metal surface.

18. **Statement 1 :** If earth suddenly stops rotating about its axis, then the value of acceleration due to gravity will become same at all the places.

Statement 2 : The value of acceleration due to gravity is independent of rotation of earth.

19. A particle moves in the $x-y$ plane under the influence of a force such that its linear momentum is $\vec{P}(t) = A[\hat{i}\cos(kt) - \hat{j}\sin(kt)]$, where A and k are constants. The angle between the force and the momentum is

- (a) 0° (b) 30° (c) 45° (d) 90°

20. A particle moves in space along the path $z = ax^3 + by^2$ in such a way that $\frac{dx}{dt} = c = \frac{dy}{dt}$ where a , b and c are constants. The acceleration of the particle is

(a) $(6ac^2x + 2bc^2)\hat{k}$ (b) $(2ax^2 + 6by^2)\hat{k}$
 (c) $(4bc^2x + 3ac^2)\hat{k}$ (d) $(bc^2x + 2by)\hat{k}$

21. The mean distance between the atoms of iron is 3×10^{-10} m and interatomic force constant for iron is 7 N m^{-1} . The Young's modulus of elasticity for iron is

(a) $2.33 \times 10^5 \text{ N m}^{-2}$ (b) $23.3 \times 10^6 \text{ N m}^{-2}$
 (c) $2.33 \times 10^9 \text{ N m}^{-2}$ (d) $2.33 \times 10^{10} \text{ N m}^{-2}$

22. A particle has two equal accelerations in two given directions. If one of the accelerations is halved, then the angle which the resultant makes with the other is also halved. The angle between the accelerations is

(a) 120° (b) 90° (c) 60° (d) 45°

23. A projectile is given an initial velocity of $\hat{i} + 2\hat{j}$. The cartesian equation of its path is (Take $g = 10 \text{ m s}^{-2}$)

(a) $y = x - 5x^2$ (b) $y = 2x - 5x^2$
 (c) $y = 2x - 15x^2$ (d) $y = 2x - 25x^2$

24. A particle is moving in a circle of radius R in such a way that at any instant the normal and tangential components of its acceleration are equal. If its speed at $t = 0$ is v_0 , the time taken to complete the first revolution is

(a) $\frac{R}{v_0}$ (b) $\frac{R}{v_0}(1 - e^{-2\pi})$
 (c) $\frac{R}{v_0}e^{-2\pi}$ (d) $\frac{2\pi R}{v_0}$

25. An electric charge of $8.85 \times 10^{-13} \text{ C}$ is placed at the centre of a sphere of radius 1 m. The electric flux through the sphere is

(a) $0.2 \text{ N C}^{-1} \text{ m}^2$ (b) $0.1 \text{ N C}^{-1} \text{ m}^2$
 (c) $0.3 \text{ N C}^{-1} \text{ m}^2$ (d) $0.01 \text{ N C}^{-1} \text{ m}^2$

26. The mass M of planet-earth is uniformly distributed over a spherical volume of radius R . Find the energy needed to disassemble the planet against the gravitational pull amongst its constituent particles.

(Given $MR = 2.5 \times 10^{31} \text{ kg m}$ and $g = 10 \text{ m s}^{-2}$)

(a) $3.0 \times 10^{32} \text{ J}$ (b) $2.5 \times 10^{32} \text{ J}$
 (c) $1.4 \times 10^{28} \text{ J}$ (d) $1.5 \times 10^{32} \text{ J}$

27. A particle is describing simple harmonic motion. If its velocities are v_1 and v_2 when the displacements

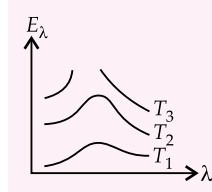
from the mean position are y_1 and y_2 respectively, then its time period is

(a) $2\pi\sqrt{\frac{y_1^2 + y_2^2}{v_1^2 + v_2^2}}$ (b) $2\pi\sqrt{\frac{v_2^2 - v_1^2}{y_1^2 - y_2^2}}$
 (c) $2\pi\sqrt{\frac{v_1^2 + v_2^2}{y_1^2 + y_2^2}}$ (d) $2\pi\sqrt{\frac{y_1^2 - y_2^2}{v_2^2 - v_1^2}}$

28. Two chambers containing m_1 g and m_2 g of a gas at pressures P_1 and P_2 respectively are put in communication with each other, temperature remaining constant. The common pressure reached will be

(a) $\frac{P_1 P_2 (m_1 + m_2)}{(P_2 m_1 + P_1 m_2)}$ (b) $\frac{P_1 P_2 m_1}{P_2 m_1 + P_1 m_2}$
 (c) $\frac{m_1 m_2 (P_1 + P_2)}{P_2 m_1 + P_1 m_2}$ (d) $\frac{m_1 m_2 P_2}{P_2 m_1 + P_1 m_2}$

29. Variation of radiant energy emitted by sun, filament of tungsten lamp and welding arc as a function of its wavelength is shown in figure. Which of the following option is the correct match?



- (a) Sun- T_1 , tungsten filament- T_2 , welding arc- T_3
 (b) Sun- T_2 , tungsten filament- T_1 , welding arc- T_3
 (c) Sun- T_3 , tungsten filament- T_1 , welding arc- T_2
 (d) Sun- T_1 , tungsten filament- T_3 , welding arc- T_2

30. A deflection magnetometer is adjusted in the usual way. When a magnet is introduced, the deflection observed is θ , and the period of oscillation of the needle in the magnetometer is T . When the magnet is removed, the period of oscillation is T_0 . The relation between T and T_0 is

(a) $T^2 = T_0^2 \cos \theta$ (b) $T = T_0 \cos \theta$
 (c) $T = \frac{T_0}{\cos \theta}$ (d) $T^2 = \frac{T_0^2}{\cos \theta}$

SOLUTIONS

1. (b): Since electric field $E = -(\text{slope of } V-x \text{ graph})$ and E inside a conductor $= 0$
 \therefore slope of $V-x$ graph between $x = d$ to $x = 2d$ should be zero.

also E in air $> E$ in dielectric

\therefore |Slope in air| $>$ |slope in dielectric|

2. (c): $K_1 = \frac{hc}{\lambda_1} - W$... (i)

and $K_2 = \frac{hc}{\lambda_2} - W$... (ii)

Here, W is work function of given metal surface.
Substituting $\lambda_1 = 2\lambda_2$ in equation (i), we get

$$\begin{aligned} K_1 &= \frac{hc}{2\lambda_2} - W \\ &= \frac{1}{2} \left(\frac{hc}{\lambda_2} \right) - W = \frac{1}{2} (K_2 + W) - W \\ \therefore K_1 &= \frac{K_2}{2} - \frac{W}{2} \quad \left(\because \frac{hc}{\lambda_2} = K_2 + W \right) \\ \Rightarrow K_1 &< \frac{K_2}{2} \end{aligned}$$

3. (c): The torque on the loop must be equal to the gravitational torque exerted about an axis tangent to the loop.

The gravitational torque

$$\tau_1 = mgr \quad \dots(i)$$

Only B_x causes a torque. Therefore torque to the magnetic field

$$\tau_2 = |\vec{M} \times \vec{B}| = MB \sin 90^\circ = \pi r^2 I B_x \quad \dots(ii)$$

Hence from equation. (i) and (ii), we get

$$\tau_1 = \tau_2 \Rightarrow mgr = \pi r^2 I B_x \quad \therefore I = \frac{mg}{\pi r B_x}$$

4. (c):
$$\frac{I_{\max}}{I_{\min}} = \left(\frac{\sqrt{I_1/I_2} + 1}{\sqrt{I_1/I_2} - 1} \right)^2 = 9$$

or
$$\frac{x+1}{x-1} = 3 \quad \left(\because x = \sqrt{I_1/I_2} \right)$$

$$\therefore x = 2$$

and
$$\frac{I_1}{I_2} = x^2 = 2^2 = 4$$

Let
$$I_1 = 4I_2$$

$$I_2 = I_0 \Rightarrow I_1 = 4I_0$$

Since
$$I_2 = I_1 \cos^2 \frac{\delta}{2} \Rightarrow I_0 = 4I_0 \cos^2 \frac{\delta}{2}$$

$$\Rightarrow \cos^2 \frac{\delta}{2} = \frac{1}{4}$$

or
$$\cos \frac{\delta}{2} = \frac{1}{2} = \cos \frac{\pi}{3} \quad \therefore \delta = \frac{2\pi}{3}$$

and
$$\delta = \frac{2\pi}{\lambda} \Delta x \text{ and } \Delta x = \frac{yd}{D}$$

$$\therefore \left(\frac{2\pi}{\lambda} \right) \left(\frac{yd}{D} \right) = \frac{2\pi}{3} \Rightarrow y = \frac{\lambda D}{3d}$$

5. (d): Since frequency $\nu \propto \sqrt{T}$
Where T is tension in sonometer wire

$$\therefore \frac{\nu_{\text{air}}}{\nu_{\text{water}}} = \sqrt{\frac{W_{\text{air}}}{W_{\text{water}}}} = \sqrt{\frac{V\rho g}{V\rho g - V\rho_w g}}$$

or
$$\frac{\nu}{\nu/2} = \sqrt{\frac{\rho}{\rho - \rho_w}}$$

or
$$2 = \sqrt{\frac{\rho}{\rho - \rho_w}}$$

or
$$4\rho - 4\rho_w = \rho \Rightarrow \rho = \frac{4}{3}\rho_w \quad \dots(i)$$

Similarly in second case

$$\frac{\nu}{\nu/3} = \sqrt{\frac{\rho}{\rho - \rho_L}}$$

or
$$3 = \sqrt{\frac{\frac{4}{3}\rho_w}{\frac{4}{3}\rho_w - \rho_L}}$$

$$= \sqrt{\frac{4}{4 - 3\frac{\rho_L}{\rho_w}}} \quad [\text{From equation (i)}]$$

Here specific gravity of the liquid $s = \frac{\rho_L}{\rho_w}$

$$\therefore 9 = \frac{4}{4 - 3s} \Rightarrow 36 - 27s = 4$$

$$\therefore s = \frac{32}{27}$$

6. (c): Since refractive index of D layer,

$$\mu = \sqrt{1 - \frac{81.45N}{v^2}}$$

Here, $N = 4000$ electrons per c.c. $= 400 \times 10^6$ electrons per m^3

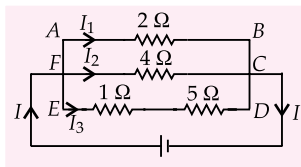
$$\nu = 200 \text{ kHz} = 200 \times 10^3 \text{ Hz}$$

$$\therefore \mu = \sqrt{1 - \frac{81.45 \times 400 \times 10^6}{(200 \times 10^3)^2}} = 0.43$$

As
$$\mu = \frac{c}{v}$$

$$\therefore \text{Phase velocity, } v = \frac{c}{\mu} = \frac{3 \times 10^8}{0.43} = 6.9 \times 10^8 \text{ m s}^{-1}$$

7. (b):



2Ω , 4Ω and $(1\Omega + 5\Omega)$ are in parallel. The potential difference across each is same.

Therefore,

$$V_{AB} = V_{FC} = V_{ED} = I_1 \times 2 = 3 \times 2 = 6 \text{ V} \quad (\because I_1 = 3 \text{ A})$$

$$I_3 = \frac{V_{ED}}{1 + 5} = \frac{6}{6} = 1 \text{ A}$$

Power dissipated in $5\ \Omega$ resistance
 $= I_3^2 \times 5 = 1^2 \times 5 = 5\ \text{W}$

8. (c): Total displacement of both charged particle due to electric field E ,

$$L = \frac{1}{2} \left(\frac{qE}{m_e} + \frac{qE}{m_p} \right) t^2$$

$$\text{or } t^2 = \frac{2L}{qE \left(\frac{1}{m_e} + \frac{1}{m_p} \right)} = \frac{2Lm_e m_p}{qE(m_e + m_p)}$$

If r is the distance travelled by proton in time t , then

$$r = \frac{1}{2} a_p t^2 = \frac{1}{2} \frac{qE}{m_p} \times \frac{2Lm_e m_p}{qE(m_e + m_p)} \left[\because a_p = \frac{qE}{m_p} \right]$$

$$= \frac{m_e L}{(m_e + m_p)}$$

9. (c): For dc voltage source, $R = \frac{V}{I} = \frac{200}{1} = 200\ \Omega$

$$\text{and } L = \frac{2\sqrt{3}}{\pi}\ \text{H}$$

When dc source is replaced by ac source,

$$\text{Impedance, } Z = \frac{E_v}{I_v} = \frac{200}{0.5} = 400\ \Omega$$

Hence the reactance of inductor

$$X_L = \sqrt{Z^2 - R^2} \quad [\because Z^2 = R^2 + X_L^2]$$

$$= \sqrt{400^2 - 200^2} = 200\sqrt{4 - 1} = 200\sqrt{3}$$

$$\text{Now, } X_L = \omega L = 2\pi\nu L = 200\sqrt{3}$$

$$\nu = \frac{200\sqrt{3}}{2\pi L} = \frac{200\sqrt{3}}{2\pi \times \frac{2\sqrt{3}}{\pi}} = 50\ \text{Hz}$$

10. (a): The temperature of ice will first increase from -10°C to 0°C . Heat supplied in this process will be

$$Q_1 = mS_i(10),$$

where m = mass of ice, and S_i = specific heat of ice
 At 0°C ice starts melting. Temperature during melting will remain constant (0°C)

Heat supplied in the process will be

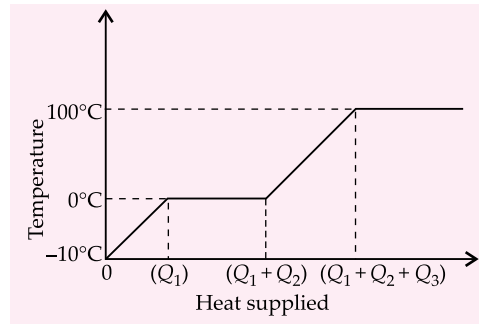
$$Q_2 = mL, \text{ where } L \text{ is latent heat of melting}$$

Now the temperature of water will increase from 0°C to 100°C . Heat supplied will be

$$Q_3 = mS_w(100)$$

where S_w is specific heat of water

Finally water at 100°C will be converted into steam and during this process temperature again remains constant. Temperature versus heat supplied graph will be as follows



11. (b): $R = R_1 + R_2 = \rho_1 \frac{l}{\pi r^2} + \rho_2 \frac{l}{\pi r^2}$
- $$R = (\rho_1 + \rho_2) \frac{l}{\pi r^2} \quad \dots(i)$$

For equivalent resistance

$$R = \rho \frac{2l}{\pi r^2} \quad \dots(ii)$$

From Equations (i) and (ii), we have

$$2\rho = \rho_1 + \rho_2$$

$$\text{or } \rho = \frac{\rho_1 + \rho_2}{2}$$

12. (a): According to the theorem of parallel axes,

$$I = I_{CM} + Ma^2$$

As a is maximum for point B.

Therefore I is maximum about B.

13. (b): By first law of thermodynamics

$$\Delta Q = \Delta U + \Delta W$$

$$= \frac{f}{2} nRT + P \int dV$$

$$= \frac{f}{2} nRT + PV$$

$$\text{or } \Delta Q = \frac{f}{2} nRT + nRT \quad (\because PV = nRT)$$

$$= \frac{3}{2} nRT + nRT = \frac{5}{2} nRT$$

$$\therefore \text{Fraction of heat energy supplied} = \frac{\Delta U}{\Delta Q}$$

$$= \frac{(3/2)nRT}{(5/2)nRT} = \frac{3}{5}$$

14. (c): From Biot-Savart Law, $\vec{B} = \frac{\mu_0}{4\pi} \frac{Id\vec{l} \times \vec{r}}{r^3}$

$$\text{For } B_1, \vec{r} = (-\hat{i} - \hat{j})$$

$$\therefore \vec{B}_1 = \frac{\mu_0}{4\pi} \frac{Idl}{2\sqrt{2}} \hat{k} \times (-\hat{i} - \hat{j}) \quad \dots(i)$$

$$\text{For } B_2, \vec{r} = \hat{i} + \hat{j}$$

$$\therefore \vec{B}_2 = \frac{\mu_0}{4\pi} \frac{Idl}{2\sqrt{2}} \hat{k} \times (\hat{i} + \hat{j}) \quad \dots(ii)$$

From (i) and (ii)

$$\vec{B}_1 = -\vec{B}_2 \text{ or } |\vec{B}_1| = |\vec{B}_2|$$

15. (b): Since resultant intensity

$$I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2} \cos\phi \quad \dots(i)$$

Applying equation (i) when phase difference is $\pi/2$

$$I_{\pi/2} = I + 4I \Rightarrow I_{\pi/2} = 5I$$

Again applying equation (i) when the phase difference is π

$$I_{\pi} = I + 4I + 2\sqrt{I}\sqrt{4I} \cos\pi$$

$$\therefore I_{\pi} = I \quad \dots(ii)$$

From equations (i) and (ii) we get,

$$I_{\pi/2} - I_{\pi} = 5I - I = 4I$$

16. (c): Since $N = N_0 e^{-\lambda t} \Rightarrow N_Y = N_0(1 - e^{-\lambda t})$

$$\text{Rate of formation of } Y, R = \frac{dN}{dt} = +\lambda N_0 e^{-\lambda t}$$

Hence the graph (c) is the correct representation.

17. (c): Both assertion and reason are true but reason is not the correct explanation of assertion. There is no emission of photoelectrons till the frequency of incident light is less than a minimum frequency, however intense light it may be. In photoelectric effect, it is a single particle collision. Intensity is $h\nu \times N$, where $h\nu$ is the individual energy of the photon and N is the total number of photon. In the wave theory, the intensity is proportional, not only to v^2 but also to the amplitude squared. For the same frequency, increase in intensity only increase the number of photons (in the quantum theory of Einstein).

18. (d): The value of g at any place is given by the relation,

$$g' = g - R_e \omega^2 \cos^2 \lambda$$

Where λ is angle of latitude and ω is the angular velocity of earth. If earth suddenly stops rotating, then $\omega = 0 \therefore g' = g$

i.e., the value of g will be same at all places.

19. (d): Here, $\vec{P}(t) = A[\hat{i} \cos(kt) - \hat{j} \sin(kt)] \quad \dots(i)$

$$\vec{F} = \frac{d\vec{P}}{dt} = Ak[-\hat{i} \sin(kt) - \hat{j} \cos(kt)] \quad \dots(ii)$$

From equation (i) and (ii)

$$\vec{F} \cdot \vec{P} = 0$$

$$\vec{F} \cdot \vec{P} = FP \cos\theta$$

$$\text{But } \vec{F} \cdot \vec{P} = 0 \Rightarrow \cos\theta = 0$$

$$\therefore \theta = 90^\circ.$$

20. (a): Given that

$$\frac{dx}{dt} = \frac{dy}{dt} = c \quad \dots(i)$$

$$\therefore \frac{d^2x}{dt^2} = \frac{d^2y}{dt^2} = 0$$

Further $z = ax^3 + by^2$

$$\therefore \frac{dz}{dt} = 3ax^2 \frac{dx}{dt} + 2by \frac{dy}{dt} = 3acx^2 + 2bcy \quad (\text{using (i)})$$

$$\text{Again, } \frac{d^2z}{dt^2} = 6acx \left(\frac{dx}{dt} \right) + 2bc \left(\frac{dy}{dt} \right) = 6ac^2x + 2bc^2$$

Now acceleration of particle is

$$\vec{a} = \frac{d^2x}{dt^2} \hat{i} + \frac{d^2y}{dt^2} \hat{j} + \frac{d^2z}{dt^2} \hat{k} = (6ac^2x + 2bc^2) \hat{k}$$

21. (d): Given, $r_0 = 3 \times 10^{-10} \text{ m}$ and $k = 7 \text{ N m}^{-1}$;

$$\therefore k = Yr_0$$

$$\text{or } Y = \frac{k}{r_0} = \frac{7}{3 \times 10^{-10}} = 2.33 \times 10^{10} \text{ N m}^{-2}$$

22. (a): As $\tan\beta = \frac{B \sin\theta}{A + B \cos\theta}$

$$= \frac{A \sin\theta}{A + A \cos\theta} \quad (\because A = B)$$

$$= \frac{\sin\theta}{1 + \cos\theta} \quad \dots(i)$$

$$\tan \frac{\beta}{2} = \frac{(A/2) \sin\theta}{A + (A/2) \cos\theta} \quad \left(\because B = \frac{A}{2} \right)$$

$$= \frac{\sin\theta}{2 + \cos\theta} \quad \dots(ii)$$

The equations are satisfied if $\theta = 120^\circ$

23. (b): Given, $\vec{u} = \hat{i} + 2\hat{j} = u_x \hat{i} + u_y \hat{j}$

$$\text{Then } u_x = 1 = u \cos\theta$$

$$\text{and } u_y = 2 = u \sin\theta$$

$$\therefore \tan\theta = \frac{u \sin\theta}{u \cos\theta} = \frac{2}{1} = 2$$

The equation of trajectory of a projectile motion is

$$y = x \tan\theta - \frac{gx^2}{2u^2 \cos^2\theta} = x \tan\theta - \frac{gx^2}{2(u \cos\theta)^2}$$

$$\therefore y = x \times 2 - \frac{10 \times x^2}{2(1)^2} = 2x - 5x^2$$

24. (b): $a_t = \frac{dv}{dt} = \frac{v^2}{R}$

$$\int_0^t \frac{dt}{R} = \int_0^v \frac{dv}{v^2} \Rightarrow t = -R \left[\frac{1}{v} \right]_{v_0}^v$$

$$v = \frac{v_0 R}{(R - v_0 t)}$$

$$\text{Now, } \frac{dr}{dt} = \frac{v_0 R}{(R - v_0 t)}$$

$$\int_0^{2\pi R} dr = v_0 R \int_0^T \frac{dt}{(R - v_0 t)}$$

$$\Rightarrow T = \frac{R}{v_0} (1 - e^{-2\pi})$$

25. (b): According to Gauss's law, the electric flux through the sphere is

$$\phi = \frac{q_{\text{in}}}{\epsilon_0} = \frac{8.85 \times 10^{-13} \text{ C}}{8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}} = 0.1 \text{ N C}^{-1} \text{ m}^2$$

26. (d): If M is the mass and R is the radius of earth, then the density $\rho = \frac{M}{\frac{4}{3}\pi R^3}$ (i)

The spherical volume may be supposed to be formed by a large number of their concentric spherical shells. Let the sphere be disassembled by removing such shells. When there is a spherical core of radius x the energy needed to disassemble a spherical shell of thickness dx is

$$dW = \frac{Gm_1 m_2}{x}, \text{ where } m_1 = \frac{4}{3}\pi x^3 \rho$$

mass of spherical shell of radius x and thickness dx is, $m_2 = 4\pi x^2 dx \rho$.

$$\therefore dW = \frac{G\left(\frac{4}{3}\pi x^3 \rho\right)(4\pi x^2 dx \rho)}{x} = \frac{16}{3}\pi^2 \rho^2 G x^4 dx$$

\therefore Total energy required

$$W = \int_0^R \frac{16}{3}\pi^2 \rho^2 G x^3 dx = \frac{16}{3}\pi^2 \rho^2 G \left[\frac{x^4}{4} \right]_0^R$$

$$= \frac{16}{3}\pi^2 \rho^2 \frac{GR^5}{5} = \frac{16}{15}\pi^2 \left(\frac{M}{\frac{4}{3}\pi R^3} \right) GR^5 \text{ (using (i))}$$

$$= \frac{3}{5} \frac{GM^2}{R}$$

But $GM = gR^2$

$$\therefore W = \frac{3}{5} \frac{gR^2}{M} \frac{M^2}{R} = \frac{3}{5} gMR = \frac{3}{5} \times 10 \times 2.5 \times 10^{31}$$

$$= 1.5 \times 10^{32} \text{ J}$$

27. (d): In simple harmonic motion,

$$\text{velocity } v = \omega \sqrt{A^2 - y^2}$$

$$\therefore v_1 = \omega \sqrt{A^2 - y_1^2} \Rightarrow v_1^2 = \omega^2 A^2 - \omega^2 y_1^2 \quad \dots (i)$$

$$\text{and } v_2 = \omega \sqrt{A^2 - y_2^2} \Rightarrow v_2^2 = \omega^2 A^2 - \omega^2 y_2^2 \quad \dots (ii)$$

Solving equations (i) and (ii), we get

$$v_2^2 - v_1^2 = \omega^2 (y_1^2 - y_2^2)$$

$$\omega = \sqrt{\frac{v_2^2 - v_1^2}{y_1^2 - y_2^2}}$$

$$\Rightarrow T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{y_1^2 - y_2^2}{v_2^2 - v_1^2}}$$

28. (a): According to Boyle's law, $PV = k$ (a constant)

$$\text{or } P \frac{m}{\rho} = k \text{ or } \rho = \frac{Pm}{k} \quad \left[\because V = \frac{m}{\rho} \right]$$

$$\text{or } \rho = \frac{P}{K} \quad \left(\because \frac{k}{m} = K \text{ a constant} \right)$$

$$\text{So, } \rho_1 = \frac{P_1}{K} \text{ and } V_1 = \frac{m_1}{\rho_1} = \frac{m_1}{P_1/K} = \frac{Km_1}{P_1}$$

$$\text{Similarly, } V_2 = \frac{Km_2}{P_2}$$

$$\therefore \text{ Total volume} = V_1 + V_2 = K \left(\frac{m_1}{P_1} + \frac{m_2}{P_2} \right)$$

Let P be the common pressure and ρ be the common density of mixture. Then

$$\rho = \frac{m_1 + m_2}{V_1 + V_2} = \frac{m_1 + m_2}{K \left(\frac{m_1}{P_1} + \frac{m_2}{P_2} \right)}$$

$$\therefore P = K\rho = \frac{m_1 + m_2}{\frac{m_1}{P_1} + \frac{m_2}{P_2}} = \frac{P_1 P_2 (m_1 + m_2)}{(m_1 P_2 + m_2 P_1)}$$

29. (c): According to Wien's displacement law, $\lambda_m T = b = \text{a constant}$. As $(\lambda_m)_3 < (\lambda_m)_2 < (\lambda_m)_1$ therefore, $T_3 > T_2 > T_1$. Hence, curve T_3 is for sun, curve T_2 is for welding arc and curve T_1 is for tungsten filament.

30. (a): In the usual setting of deflection magnetometer, field due to magnet (F) and horizontal component (H) of earth's field are perpendicular to each other. Therefore, the net field on the magnetic needle is $\sqrt{F^2 + H^2}$

$$\therefore T = 2\pi \sqrt{\frac{I}{M\sqrt{F^2 + H^2}}} \quad \dots (i)$$

When the magnet is removed,

$$T_0 = 2\pi \sqrt{\frac{I}{MH}} \quad \dots (ii)$$

$$\text{Also, } \frac{F}{H} = \tan \theta$$

Dividing (i) by (ii), we get

$$\frac{T}{T_0} = \sqrt{\frac{H}{\sqrt{F^2 + H^2}}}$$

$$= \sqrt{\frac{H}{\sqrt{H^2 \tan^2 \theta + H^2}}} = \sqrt{\frac{H}{H\sqrt{\sec^2 \theta}}} = \sqrt{\cos \theta}$$

$$\Rightarrow \frac{T^2}{T_0^2} = \cos \theta \quad \therefore T^2 = T_0^2 \cos \theta$$

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UNIT-7

Optics | Modern Physics

OPTICS

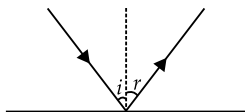
Optics is the branch of physics which deals with the study of production, propagation and nature of light. It is divided into two branches :

(i) Ray optics (ii) Wave optics

REFLECTION OF LIGHT

It is defined as, a part of incident light is turned back into the same medium.

In figure i and r represent incident ray and reflected ray respectively.



Laws of Reflection

- The angle of incidence i equals the angle of reflection r .
- Incident ray, the normal and the reflected ray lie in the same plane.

The above laws of reflection are valid both in case of plane and curved reflecting surfaces.

For normal incidence *i.e.*, $\angle i = 0$, $\angle r = 0$. Hence a ray of light falling normally on a mirror retraces its path on reflection.

Reflection from Plane Surface

- The image formed by a plane mirror is at the same distance behind the mirror as the object is in front of it.
- The image formed by a plane mirror is laterally inverted. The lateral inversion means that the right side of the object appears as the left side of the image and vice-versa.
- The image formed by a plane mirror is virtual, erect w.r.t. object and of the same size as the object.

- If keeping the incident ray fixed, the plane mirror is rotated through an angle θ , the reflected ray turns through double the angle *i.e.*, 2θ in that very direction.
- If the object is fixed and the mirror moves relative to the object with a speed v , the image moves with a speed $2v$ relative to the object.
- If the mirror is fixed and the object moves relative to the mirror with a speed v , the image also moves with the same speed v relative to the mirror.
- Deviation suffered by a light ray incident at an angle i is given by

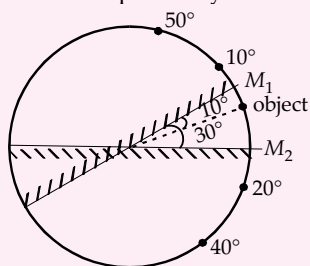
$$\delta = (180^\circ - 2i)$$

Number of Images Formed by Two Inclined Mirrors

- If $\frac{360^\circ}{\theta} = \text{even number}$; number of images $= \frac{360^\circ}{\theta} - 1$
- If $\frac{360^\circ}{\theta} = \text{odd number}$; number of images $= \frac{360^\circ}{\theta} - 1$ if the object is placed on the angle bisector.
- If $\frac{360^\circ}{\theta} = \text{odd number}$; number of images $= \frac{360^\circ}{\theta}$, if the object is not placed on the angle bisector.
- If $\frac{360^\circ}{\theta} \neq \text{integer}$, then count the number of images as explained above.

Illustration 1 : Two mirrors are inclined by an angle 30° . An object is placed making 10° with the mirror M_1 . Find the positions of first two images formed by each mirror. Find the total number of images.

Soln.: Figure is self explanatory.



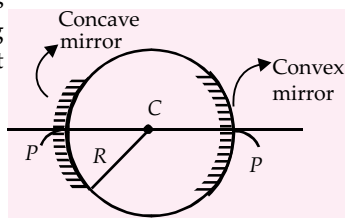
$$\text{Number of images} = \frac{360^\circ}{30^\circ} = 12 (\text{even number})$$

$$\therefore \text{number of images} = 12 - 1 = 11$$

SPHERICAL MIRRORS

Spherical Mirror is formed by polishing one surface of a part of sphere.

Depending upon which part is shining the spherical mirror is classified as

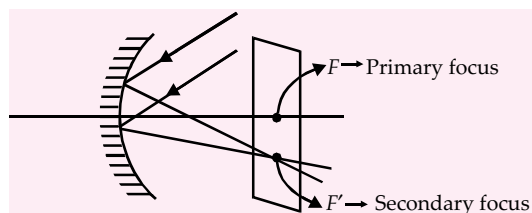


- **Concave mirror**, if the side towards center of curvature is shining.
- **Convex mirror**, if the side away from the center of curvature is shining.

Important Terms for Spherical Mirrors

- **Pole (P)**, is the mid point of reflecting surface.
- **Centre of curvature (C)**, is the centre of the sphere of which the mirror is a part.
- **Radius of curvature**, is the radius of the sphere of which the mirror is a part. Distance between P and C.
- **Principal axis**, is the straight line connecting pole P and centre of curvature C.
- **Principal focus (F)**, is the point of intersection of all the reflected rays which strikes the mirror parallel to the principal axis. In concave mirror it is real and in the convex mirror it is virtual.
- **Focal length (f)**, is distance from pole to focus.
- **Aperture**, the diameter of the mirror is called aperture of the mirror.

Focal Plane : Plane perpendicular to principal axis and passing through focus is known as focal plane.



Sign convention

We follow Cartesian co-ordinate system conventions according to which

- The pole of mirror is the origin.
- The distance measured in the direction of the incident rays is considered as positive x-axis.
- The heights measured in the vertically up direction are positive y-axis.

Mirror Formula

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

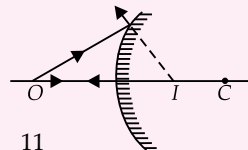
u = distance of object, v = image distance, f = focal length and $f = R/2$; R = radius of curvature.

Illustration 2 : A convex mirror has its radius of curvature 20 cm. Find the position of the image of an object placed at a distance of 12 cm from the mirror.

Soln.: The situation is shown in figure.

Here $u = -12$ cm and $R = +20$ cm. We have,

$$\begin{aligned} \frac{1}{u} + \frac{1}{v} &= \frac{2}{R} \\ \text{or } \frac{1}{v} &= \frac{2}{R} - \frac{1}{u} \\ &= \frac{2}{20 \text{ cm}} - \frac{1}{-12 \text{ cm}} = \frac{11}{60 \text{ cm}} \\ \text{or, } v &= \frac{60}{11} \text{ cm.} \end{aligned}$$

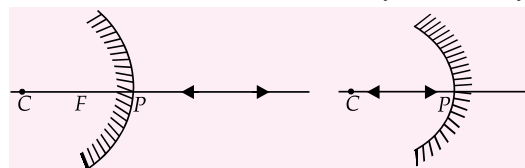


The positive sign of v shows that the image is formed on the right side of the mirror. It is a virtual image.

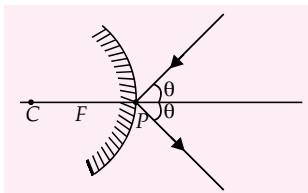
Ray Tracing

Following facts are useful in ray tracing.

- If the incident ray is parallel to the principal axis, the reflected ray passes through the focus.
- If the incident ray passes through the focus, then the reflected ray is parallel to the principal axis.
- Incident ray passing through centre of curvature will be reflected back through the centre of curvature (because it is a normally incident ray).



- It is easy to make the ray tracing of a ray incident at the pole as shown below.



Magnification

- Linear magnification : $m = \frac{h_2}{h_1} = -\frac{v}{u}$

h_1 = height of the object, h_2 = height of the image. (h_1 and h_2 both are perpendicular to the principal axis of mirror)

- Note:** If the image is upright or erect with respect to the object then m is positive. And m is negative if the image is inverted with respect to the object.

- Longitudinal magnification = $\frac{v_2 - v_1}{u_2 - u_1}$

[for small sized object]

REFRACTION OF LIGHT

When light passes obliquely from one transparent medium to another, the direction of its path changes at the interface of the two medium. This phenomenon is known as refraction of light.

If a ray of light passes from an optically rarer medium to a denser medium, it bends towards the normal (i.e., $\angle r < \angle i$).

If a ray of light passes from an optically denser medium to a rarer medium, it bends away from the normal (i.e., $\angle r > \angle i$).

A ray of light travelling along the normal passes undeflected, the incident ray and refracted ray make zero angle with normal (i.e., $\angle i = \angle r = 0^\circ$)

Laws of Refraction

- The incident ray, the normal to the interface at the point of incidence and the refracted ray all lie in the same plane.
- The ratio of the sine of the angle of incidence to the sine of the angle of refraction is always a constant (a different constant for a different set of media).

$$\text{i.e. } \frac{\sin i}{\sin r} = \text{constant} = {}^1\mu_2$$

This constant (${}^1\mu_2$) is called refractive index of medium 2 (in which refracted ray travels) w.r. to medium 1 (in which incident ray travels). It is known as Snell's law and holds good for all angles of incidence.

Refractive index

The refractive index of a medium for a light of given wavelength may be defined as the ratio of the speed of light in vacuum to its speed in that medium.

$$\mu = \frac{\text{Speed of light in vacuum}}{\text{Speed of light in medium}} = \frac{c}{v}$$

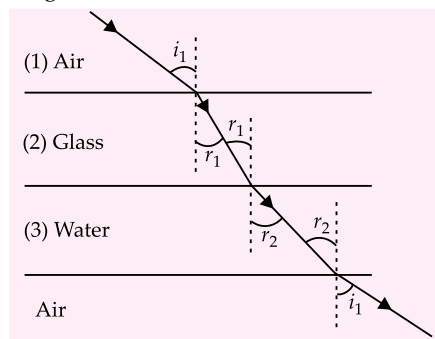
Also, ${}^{\text{vacuum}}\mu_{\text{medium}}$ (refractive index of medium w.r. to vacuum) = $\frac{c}{v}$

Refractive index of a medium with respect to vacuum is also called **absolute refractive index**.

- When a light ray travels from one medium to another, its frequency remains constant but its wavelength as well as velocity changes.
- The deviation of the incident ray when it is refracted is given by an angle $\delta = |i - r|$.

Refraction Through Various Medium

If a light ray passes through a number of parallel media and if the first and the last medium are same, the emergent ray is parallel to the incident ray as shown in figure below.



$${}^1\mu_2 = \frac{\sin i_1}{\sin r_1}$$

$${}^2\mu_3 = \frac{\sin r_1}{\sin r_2} \text{ and } {}^3\mu_1 = \frac{\sin r_2}{\sin i_1}$$

$$\text{Hence, } {}^1\mu_2 \times {}^2\mu_3 \times {}^3\mu_1 = \frac{\sin i_1}{\sin r_1} \times \frac{\sin r_1}{\sin r_2} \times \frac{\sin r_2}{\sin i_1} = 1$$

Principle of reversibility

$${}^1\mu_2 = \frac{1}{{}^2\mu_1}$$

Illustration 3 : The refractive index of glass is 1.5 and that of water is 1.3. If the speed of light in water is $2.25 \times 10^8 \text{ m s}^{-1}$, what is the speed of light in glass?

Soln.: Here, ${}^a\mu_g = \frac{c}{v_g} = 1.5$, ${}^a\mu_w = \frac{c}{v_w} = 1.3$

$$\therefore \frac{c}{v_w} \times \frac{v_g}{c} = \frac{1.3}{1.5}$$

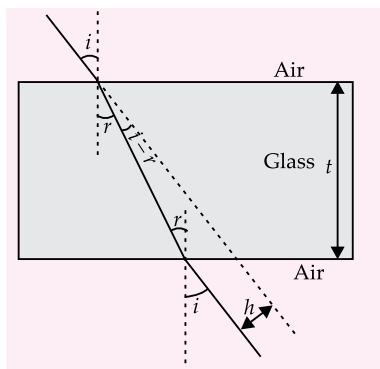
$$\text{or } v_g = \frac{1.3}{1.5} \times v_w = \frac{1.3}{1.5} \times 2.25 \times 10^8 \\ = 1.95 \times 10^8 \text{ m s}^{-1}.$$

LATERAL SHIFT DUE TO GLASS SLAB

When the medium is same on both sides of a glass slab, then the deviation of the emergent ray is zero. That is the emergent ray is parallel to the incident ray but it does suffer lateral displacement/shift with respect to the incident ray and is given by

$$\text{Lateral shift, } h = t \frac{\sin(i - r)}{\cos r}$$

where t is the thickness of the slab.



Real depth and apparent depth

An object placed in a denser medium (e.g. water), when viewed from a rarer medium (e.g. air) appears to be at a lesser depth than its real depth. This is on account of refraction of light.

$$\text{Refractive index} = \frac{\text{Real depth}}{\text{Apparent depth}}$$

$$\text{or apparent depth} = \frac{\text{Real depth}}{\text{Refractive index}}$$

As the refractive index of any medium (other than vacuum) is greater than unity, so the apparent depth is less than the real depth.

The height through which an object appears to be raised in a denser medium is called normal shift or apparent shift.

Normal shift = real depth – apparent depth

$$d = t - \frac{t}{\mu} = t \left(1 - \frac{1}{\mu} \right)$$

Here, t is the real depth of water and μ is its refractive index.

Therefore, the normal shift in the position of an object, when seen through a denser medium depends on the refractive index of the medium. The higher the value of μ , greater is the apparent shift d .

Illustration 4 : A mark is made on the bottom of beaker and a microscope is focussed on it. The microscope is raised through 1.5 cm. To what height water must be poured into the beaker to bring the mark again into focus? ($\mu_{\text{water}} = \frac{4}{3}$)

Soln.: Here, apparent shift, $d = 1.5$ cm

Let t be the height through which water must be poured into the beaker.

$$d = t \left(1 - \frac{1}{\mu} \right)$$

$$1.5 = t \left(1 - \frac{1}{4/3} \right)$$

$$t = 1.5 \times 4 = 6.0 \text{ cm}$$

TOTAL INTERNAL REFLECTION

The total internal reflection is the phenomenon in which a ray of light travelling from an optically denser into an optically rarer medium at an angle of incidence greater than the critical angle for the two media is totally reflected back into the same medium.

Necessary conditions for total internal reflection

- Light is travelling from optically denser to optically rarer medium.
- The angle of incidence at the surface is greater than the critical angle for the pair of media.

Critical angle

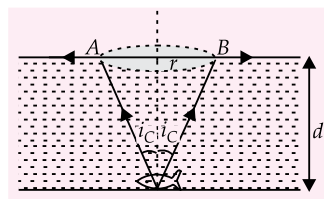
The critical angle between the two media is the angle of incidence in the optically denser medium for which the angle of refraction is 90° . It is given by

$$\sin i_c = \frac{1}{\mu}$$

Let i = angle of incidence. Consider the following cases :

- If $i < i_c$, then refraction takes place.
- If $i = i_c$, then grazing emergence takes place.
- If $i > i_c$, then total internal reflection takes place.
- Critical angle depends on nature of media in contact and also on the wavelength of light.
- Critical angle for red light is more than that for blue light.
- A fish in water at a depth d sees the world outside through a horizontal circle of radius

$$r = d \tan i_c = \frac{d}{\sqrt{\mu^2 - 1}}$$



Applications of total internal reflection

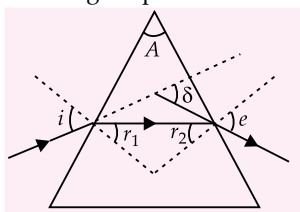
- Brilliance of diamonds
- Mirage
- Totally reflecting glass prisms
- Optical fibres

REFRACTION THROUGH A PRISM

A prism is a homogeneous, transparent medium bounded by two plane surfaces inclined at an angle A with each other. These surfaces are called the refracting surfaces and angle between them is called the refracting angle or the angle of prism A .

The angle between the incident ray and the emergent ray is called the angle of deviation.

For refraction through a prism it is found that



$$\delta = i + e - A \text{ where } A = r_1 + r_2$$

When A and i are small

$$\therefore \delta = (\mu - 1) A$$

In a position of minimum deviation $\delta = \delta_m$, $i = e$, and $r_1 = r_2 = r$

$$\therefore i = \left(\frac{A + \delta_m}{2} \right) \text{ and } r = \frac{A}{2}$$

The refractive index of the material of the prism is

$$\mu = \frac{\sin \left[\frac{(A + \delta_m)}{2} \right]}{\sin \left(\frac{A}{2} \right)}$$

This is called prism formula.

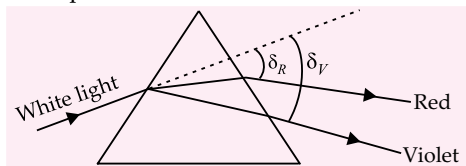
Dispersion of light

It is the phenomenon of splitting of white light into its constituent colours on passing through a prism. This is because different colours have different wavelengths ($\lambda_R > \lambda_V$). According to Cauchy's formula

$$\mu = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4}$$

where A, B, C are arbitrary constants. Therefore, μ of material of prism for different colours is different ($\mu_V > \mu_R$). As $\delta = (\mu - 1) A$, therefore different colours turn through different angles on passing through the prism. This is the cause of dispersion.

Note : Vacuum is a non-dispersive medium whereas glass is a dispersive medium.



Angular dispersion, $\theta = \delta_V - \delta_R = (\mu_V - \mu_R) A$

where μ_V and μ_R are the refractive indices for violet and red light respectively.

Dispersive power, $\omega = \frac{\text{angular dispersion}}{\text{mean deviation}}$

$$\omega = \frac{\mu_V - \mu_R}{(\mu - 1)}$$

where $\mu = \frac{\mu_V + \mu_R}{2} = \text{mean refractive index}$

The dispersive power depends on the material of the prism.

Dispersive power is a unit less and dimensionless quantity.

Dispersive power of a flint glass prism is more than that of a crown glass prism.

Combination of Prisms

When two prisms are combined together, we can get deviation without dispersion or vice versa.

Deviation without Dispersion

Condition for deviation without dispersion is

$$\theta + \theta' = 0$$

$$(\mu_V - \mu_R) A + (\mu'_V - \mu'_R) A' = 0$$

$$\text{or } A' = - \frac{(\mu_V - \mu_R) A}{(\mu'_V - \mu'_R)}$$

–ve sign shows that the refracting angles of the two prisms are in opposite direction.

where A and A' are refracting angles of two prisms respectively and μ_V, μ_R and μ'_V, μ'_R be the refractive indices of the violet and red light of the corresponding prisms.

Under this condition, net deviation produced by the combination is

$$= \delta + \delta' = (\mu - 1) A + (\mu' - 1) A'$$

The prism which produces deviation without dispersion is called **achromatic prism**.

Dispersion without Deviation

Condition for dispersion without deviation is

$$\delta + \delta' = 0$$

$$(\mu - 1) A + (\mu' - 1) A' = 0$$

$$\text{or } A' = - \frac{(\mu - 1) A}{(\mu' - 1)}$$

–ve sign shows that the refracting angles of two prisms are in opposite direction.

where μ and μ' be the refractive indices of the material of two prisms respectively.

Under this condition, net angular dispersion produced by the combination is

$$= (\delta_V - \delta_R) + (\delta'_V - \delta'_R)$$

$$= (\mu_V - \mu_R) A + (\mu'_V - \mu'_R) A'$$

The prism which produces dispersion with deviation is called **direct vision prism**.

SCATTERING OF LIGHT

As sunlight travels through the earth's atmosphere, it gets scattered (changes its direction) by the atmospheric particles. Light of shorter wavelengths is scattered much more than the light of longer wavelengths. The amount of scattering is inversely proportional to the fourth power of the wavelength. This is called Rayleigh scattering.

Illustrations of Scattering of Light

- Blue colour of sky
- White colour clouds
- The sun looks reddish at the time of sun rise and sun set

Illustration 5 : A prism is made of glass of unknown refractive index. A parallel beam of light is incident on a face of the prism. By rotating the prism, the minimum angle of deviation is measured to be 40° . What is the refractive index of the prism? If the prism is placed in water (refractive index 1.33), predict the new angle of minimum deviation of a parallel beam of light. The refracting angle of the prism is 60° .

Soln.: Here; as $\delta_m = 40^\circ$, $A = 60^\circ$,

$$\therefore {}^a\mu_g = \frac{\sin[(A + \delta_m)/2]}{\sin(A/2)} = \frac{\sin 50^\circ}{\sin 30^\circ} = \frac{0.7660}{0.5000} = 1.53$$

When the prism is placed in water, refractive index of prism (i.e., glass) w.r.t. water i.e.,

$${}^w\mu_g = \frac{{}^a\mu_g}{{}^a\mu_w} = \frac{1.53}{1.33} = 1.1519 \quad (\because {}^a\mu_w = 1.33)$$

Let δ'_m be the new minimum angle of deviation. Clearly,

$${}^w\mu_g = \frac{\sin[(A + \delta'_m)/2]}{\sin(A/2)}$$

$$\text{or } 1.152 = \frac{\sin[(A + \delta'_m)/2]}{\sin 30^\circ}$$

$$\text{or } \sin\left[\frac{(A + \delta'_m)}{2}\right] = 1.152 \times \sin 30^\circ$$

$$= 1.152 \times 0.5000 = 0.5759$$

$$\text{or } \frac{(A + \delta'_m)}{2} = 35^\circ 10'$$

$$\text{or } \delta'_m = 2 \times 35^\circ 10' - 60^\circ = 10^\circ 20'$$

Illustration 6 : Calculate the angular dispersion between violet and red colours produced by a small angled prism with vertex angle 5° . Refractive index of the material of the prism for violet = 1.66 and that for red = 1.54.

Soln.: With usual notation, we are given that,

$$A = 5^\circ, \mu_v = 1.66 \text{ and } \mu_r = 1.54$$

$$\text{Angular dispersion} = (\delta_v - \delta_r) = (\mu_v - \mu_r)A$$

$$= (1.66 - 1.54) \times 5^\circ = 0.12 \times 5^\circ = 0.6^\circ$$

REFRACTION AT SPHERICAL SURFACES

A refractive surface which forms a part of a sphere of transparent medium is called a spherical refracting surface. Spherical refracting surfaces are of two types

- Convex spherical refracting surface
- Concave spherical refracting surface.

For both surfaces refracting formula is given by

$$\frac{{}_1\mu_2}{v} - \frac{1}{u} = \frac{{}_1\mu_2 - 1}{R}$$

where ${}_1\mu_2$ is refractive index of second medium with respect to first and u , v , R are the object distance, image distance and radius of curvature of the spherical surface respectively.

If μ_1 and μ_2 are refractive indices of first and second medium with respect to air, then

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

Illustration 7 : What curvature must be given to the bounding surface of a refracting medium ($\mu = 1.5$) for the virtual image of an object in the adjacent medium ($\mu = 1$) at 10 cm to be formed at a distance of 40 cm?

Soln.: Let R be the radius of curvature of the refracting surface. As the object lies in the rarer medium and the image formed is virtual,

$$\therefore -\frac{\mu_1}{u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$

Here $\mu_1 = 1$, $\mu_2 = 1.5$, $u = -10$ cm, $v = -40$ cm

$$\therefore \frac{-1}{-10} + \frac{1.5}{-40} = \frac{1.5 - 1}{R}$$

$$\text{or } \frac{0.5}{R} = \frac{1}{10} - \frac{1.5}{40} = \frac{2.5}{40}$$

$$\therefore R = \frac{40 \times 0.5}{2.5} = +8 \text{ cm}$$

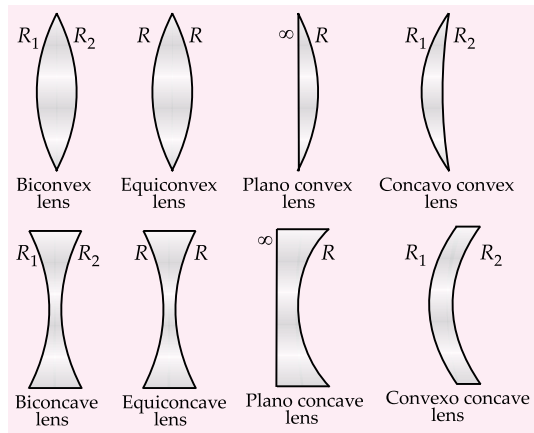
LENSES

A lens is a transparent medium bounded by two refracting surfaces such that at least one of the refracting surfaces is curved.

If the thickness of the lens is negligibly small in comparison to the object distance or the image distance, the lens is called thin. Here we shall limit ourselves to thin lenses.

Types of Lenses

Broadly, lenses are of the following types :

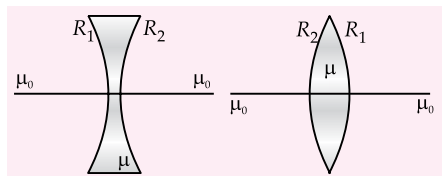


Lens Maker's Formula

The focal length (f) of a lens depends upon the refractive indices of the material of the lens and the medium in which the lens is present and the radii of curvature of both sides. The following relation giving focal length (f) is called as 'lens maker's formula'.

$$\frac{1}{f} = \left(\frac{\mu}{\mu_0} - 1 \right) \left[\frac{1}{R_1} - \frac{1}{R_2} \right]$$

where μ = refractive index of the material of the lens,
 μ_0 = refractive index of the medium.



Lens Formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Linear Magnification (m)

The ratio of the size of the image formed by a lens to the size of the object is called linear magnification produced by the lens. It is denoted by m .

If O and I are the sizes of the object and image respectively, then

$$m = \frac{I}{O} = \frac{v}{u}$$

Illustration 8 : The radii of curvature of a double convex lens are 15 cm and 30 cm and its refractive index is 1.5. Find its focal length.

Soln.: $\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$

Here, $R_1 = 15$ cm, $R_2 = -30$ cm

$$\mu = 1.5$$

$$= (1.5 - 1) \left(\frac{1}{15} - \frac{1}{-30} \right)$$

$$= 0.5 \times \left(\frac{1}{15} + \frac{1}{30} \right) = 20 \text{ cm}$$

Power of a Lens

The power of a lens is defined as the reciprocal of the focal length in metre.

$$P = \frac{1}{f(\text{in m})}$$

The SI unit of power of lens is diopter (D).

For a convex lens, P is positive.

For a concave lens, P is negative.

When focal length (f) of lens is in cm, then

$$P = \frac{100}{f(\text{in cm})}, \text{ dioptre.}$$

Combination of Thin Lenses in Contact

When a number of thin lenses of focal length f_1, f_2, \dots etc. are placed in contact coaxially, the equivalent focal length F of the combination is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \dots$$

The total power of the combination is given by

$$P = P_1 + P_2 + P_3 + \dots$$

The total magnification of the combination is given by

$$m = m_1 \times m_2 \times m_3 \dots$$

When two thin lenses of focal lengths f_1 and f_2 are placed coaxially and separated by a distance d , the focal length of combination is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

In terms of power, $P = P_1 + P_2 - dP_1P_2$.

Silvering of a Lens

Let a planoconvex lens is having a curved surface of radius of curvature R and has refractive index μ . If its plane surface is silvered, it behaves as a concave mirror of focal length

$$f = -\frac{R}{2(\mu - 1)}$$

If the curved surface of planoconvex lens is silvered then it behaves as a concave mirror of focal length

$$f = -\frac{R}{2\mu}$$

Displacement Method

For a convex lens, the minimum distance between the object and its real image is $4f$. If a convex lens is placed between an object O and a screen S such that the distance $OS \geq 4f$, there are two positions of the lens which give a sharp image on the screen.

The focal length of the lens is given by

$$f = \frac{D^2 - d^2}{4D}$$

where D = distance between the screen and the object,
 d = distance between the two positions of the lens.

If I_1, I_2 are the two sizes of image of the object of size O , then $O = \sqrt{I_1 I_2}$

OPTICAL INSTRUMENTS

Simple Microscope

It is also called magnifying glass or simple magnifier. It consists of a converging lens of small focal length.

Magnifying power of a simple microscope

When the image is formed at infinity (far point),

$$M = \frac{D}{f}$$

where f is the focal length of convex lens.

When the image is formed at the least distance of distinct vision D (near point),

$$M = 1 + \frac{D}{f}$$

Compound Microscope

It consists of two convergent lenses of short focal lengths and apertures arranged coaxially. Lens facing the object is called objective or field lens while the lens facing the eye, is called eye-piece or ocular. The objective has a smaller aperture and smaller focal length than eye-piece.

Magnifying power of a compound microscope

When the final image is formed at infinity (normal adjustment),

$$M = -\frac{v_o}{u_o} \left(\frac{D}{f_e} \right)$$

Length of tube, $L = v_o + f_e$

When the final image is formed at least distance of distinct vision,

$$M = -\frac{v_o}{u_o} \left(1 + \frac{D}{f_e} \right)$$

where u_o and v_o represent the distance of object and intermediate image from the objective lens, f_e is the focal length of an eye lens.

Length of the tube, $L = v_o + \left(\frac{f_e D}{f_e + D} \right)$

Astronomical Telescope (Refracting Type)

It consists of two converging lenses. The one facing the object is called objective or field lens and has large focal length and aperture while the other facing the eye is called eye-piece or ocular has small focal length and aperture.

Magnifying power of an astronomical telescope

When the final image is formed at infinity (normal adjustment),

$$M = -\frac{f_o}{f_e}$$

Length of tube, $L = f_o + f_e$

When the final image is formed at least distance of distinct vision,

$$M = -\frac{f_o}{f_e} \left(1 + \frac{f_e}{D} \right)$$

Length of tube, $L = f_o + \frac{f_e D}{f_e + D}$

Terrestrial Telescope

It is used for observing far off objects on the ground. The essential requirement of such a telescope is that final image must be erect w.r.t. the object. To achieve it, an inverting convex lens (of focal length f) is used in between the objective and eye piece of astronomical telescope. This lens is known as erecting lens.

In normal adjustment,

Magnifying power, $M = \frac{f_o}{f_e}$

Length of the tube, $L = f_o + 4f + f_e$

Galileo's Terrestrial Telescope

It consists of an objective which is a convex lens of large focal length and an eye-piece which is a concave lens of short focal length ($f_o > f_e$).

In the normal adjustment,

Magnifying power, $M = \frac{f_o}{f_e}$

Length of the tube, $L = f_o - f_e$

WAVEFRONT

A wavefront is defined as the continuous locus of all such particles of the medium which are vibrating in the same phase of any instant.

Types of Wavefront

The geometrical shape of a wavefront depends on the source of disturbance. Some of the common shapes are

- **Spherical wavefront** : In the case of waves travelling in all directions from a point source, the wavefronts are spherical in shape.
- **Cylindrical wavefront** : If the source of light is linear in shape, such as a fine rectangular slit, the wavefront is cylindrical in shape.
- **Plane wavefront** : As a spherical or cylindrical wavefront advances, its curvature decreases progressively.

HUYGEN'S PRINCIPLE

According to Huygen's principle, each point on a wavefront is a source of secondary waves, which add up to give a wavefront at any later time.

Assumptions

- The secondary wavelets spread out in all directions with the speed of light in the given medium.
- The new wavefront at any later time is given by the forward envelope (tangential surface) in the forward direction of the secondary wavelets at that time.
- Each point on a wavefront acts as fresh source of new disturbance called secondary waves or wavelets.

COHERENT SOURCES

Two sources of light which continuously emit light waves of same frequency (or wavelength) with a zero or constant phase difference between them, are called coherent sources.

Conditions for obtaining two coherent sources of light

- The two sources of light must be obtained from a single source by some method.
- The two sources must give monochromatic light.
- The path difference between the waves arriving on the screen from the two sources must not be large
- Two independent sources cannot be coherent.

SUPERPOSITION OF WAVES

When a number of waves travelling through a medium, superpose on each other, the resultant displacement at any point at a given instant is equal to the vector sum of displacements due to the individual waves at that point.

Interference of Light

If two light waves of the same frequency and having zero or constant phase difference travelling in the same direction super position gets redistributed becoming maximum at some points and minimum at others. This phenomenon is called interference of light.

Intensity Distribution

If A_1, A_2 are the amplitudes of interfering waves due to two coherent sources and ϕ is constant phase difference between the two waves at any point P , then The resultant amplitude at P will be

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos \phi}$$

Resultant intensity at P is

$$I = I_1 + I_2 + 2\sqrt{I_1I_2} \cos \phi$$

When $\phi = 2n\pi$, where $n = 0, 1, 2, \dots$

Then, $A = A_{\max} = (A_1 + A_2)$

$$I = I_{\max} = I_1 + I_2 + 2\sqrt{I_1I_2} = (\sqrt{I_1} + \sqrt{I_2})^2$$

When $\phi = (2n - 1)\pi$, where $n = 1, 2, 3, \dots$

Then, $A = A_{\min} = (A_1 - A_2)$

$$I = I_{\min} = I_1 + I_2 - 2\sqrt{I_1I_2} = (\sqrt{I_1} - \sqrt{I_2})^2$$

$$\frac{I_{\max}}{I_{\min}} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2}$$

If the amplitude of the two waves are equal $A_1 = A_2 = A_0$, then resultant amplitude

$$A = \sqrt{2A_0^2 + 2A_0^2 \cos \phi} = 2A_0 \cos\left(\frac{\phi}{2}\right)$$

Resultant intensity, $I = 4I_0 \cos^2\left(\frac{\phi}{2}\right)$

In this case, $A_{\max} = 2A_0$, $I_{\max} = 4I_0$

$$A_{\min} = 0, I_{\min} = 0$$

Note : If the sources are incoherent, $I = I_1 + I_2$.

YOUNG'S DOUBLE SLIT EXPERIMENT

The phenomenon of interference of light was first observed by a British Physicist Thomas Young. Using two slits illuminated by monochromatic light source, he obtained alternately bright and dark band on the screen. These bands are called interference fringes or interference bands.

For constructive interference (i.e. formation of bright fringes)

For n^{th} bright fringe,

Path difference = $n\lambda$

where $n = 0$ for central bright fringe

$n = 1$ for first bright fringe,

$n = 2$ for second bright fringe and so on

d = distance between the two slits

D = distance of slits from the screen

The position of n^{th} bright fringe from the centre of the screen is given by

$$x_n = n\lambda \frac{D}{d}$$

For destructive interference (i.e. formation of dark fringes)

For n^{th} dark fringe,

$$\text{path difference} = (2n - 1) \frac{\lambda}{2}$$

where

$n = 1$ for first dark fringe,

$n = 2$ for second dark fringe and so on.

The position of n^{th} dark fringe from the centre of the screen is given by

$$\therefore x_n = (2n - 1) \frac{\lambda}{2} \frac{D}{d}$$

Fringe width : The distance between any two consecutive bright or dark fringes is called fringe width.

$$\text{Fringe width, } \beta = \frac{\lambda D}{d}$$

$$\text{Angular fringe width, } \theta = \frac{\beta}{D} = \frac{\lambda}{d}$$

If W_1, W_2 are widths of two slits, I_1, I_2 are intensities of light coming from two slits; A_1, A_2 are the amplitudes of light from these slits, then

$$\frac{W_1}{W_2} = \frac{I_1}{I_2} = \frac{A_1^2}{A_2^2}$$

$$\frac{I_{\max}}{I_{\min}} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2}$$

$$\text{Fringe visibility } V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

When entire apparatus of Young's double slit experiment is immersed in a medium of refractive index μ , then fringe width becomes

$$\beta' = \frac{\lambda' D}{\mu d} = \frac{\beta}{\mu}$$

When a thin transparent plate of thickness t and refractive index μ is placed in the path of one of the interfering waves, fringe width remains unaffected but the entire pattern shifts by

$$\Delta x = (\mu - 1) t \frac{D}{d} = (\mu - 1) t \frac{\beta}{\lambda}$$

This shifting is towards the side in which transparent plate is introduced.

Illustration 9 : Two sources of intensity I and $4I$ are used in an interference experiment. Find the intensity at points where the waves from two sources superimpose with a phase difference (i) zero (ii) $\pi/2$ and (iii) π .

Soln.: The resultant intensity at a point where phase difference is ϕ is

$$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$$

As $I_1 = I$ and $I_2 = 4I$, therefore

$$I_R = I_1 + 4I + 2\sqrt{I \cdot 4I} \cos \phi = 5I + 4I \cos \phi$$

(i) When $\phi = 0$, $I_R = 5I + 4I \cos 0 = 9I$.

(ii) When $\phi = \frac{\pi}{2}$, $I_R = 5I + 4I \cos \frac{\pi}{2} = 5I$.

(iii) When $\phi = \pi$, $I_R = 5I + 4I \cos \pi = 5I - 4I = I$.

Illustration 10 : In a Young's double slit experiment, the slits are separated by 0.5 mm and screen is placed 1.0 m away. It is found that the ninth bright fringe is at a distance of 8.835 mm from the second dark fringe. Find the wavelength of light used.

Soln.: The distance of n^{th} bright fringe from the central bright fringe is

$$x_n = \frac{nD\lambda}{d} = n\beta \quad \therefore x_9 = 9\beta$$

The distance of n^{th} dark fringe from the central bright fringe is

$$x'_n = (2n - 1) \frac{D\lambda}{2d} = (2n - 1) \frac{\beta}{2}$$

$$\therefore x'_2 = \frac{3}{2}\beta$$

$$\text{But } x_9 - x'_2 = 8.835 \text{ mm} \quad [\text{Given}]$$

$$\text{or } 9\beta - \frac{3}{2}\beta = 8.835 \text{ mm} \quad \text{or } \frac{15}{2}\beta = 8.835 \text{ mm}$$

$$\text{or } \beta = \frac{8.835 \times 2}{15} = 1.178 \text{ mm} = 1.178 \times 10^{-3} \text{ m}$$

$$\text{Hence, } \lambda = \frac{\beta d}{D} = \frac{1.178 \times 10^{-3} \times 0.5 \times 10^{-3}}{1.0} \text{ m}$$

$$= 0.5890 \times 10^{-6} \text{ m} = 5898 \text{ \AA}$$

Interference in thin films

A thin film of liquid (e.g. soap film or a layer of oil over water) appears bright or dark when viewed in monochromatic light. This effect is caused due to the interference of light reflected from the top and bottom faces of the film.

Interference in reflected light (reflected system of light)

For a bright fringe,

$$2\mu t \cos r = (2n + 1) \frac{\lambda}{2}$$

where, $n = 0, 1, 2, 3, \dots$

For a dark fringe

$$2\mu t \cos r = n\lambda$$

where, $n = 0, 1, 2, 3, \dots$

Interference in transmitted light (transmitted system of light)

For a bright-fringe,

$$2\mu t \cos r = n\lambda$$

For a dark fringe,

$$2\mu t \cos r = (2n + 1) \frac{\lambda}{2}$$

Hence, the condition for maxima and minima in the reflected system are just opposite to those for the transmitted system. Thus the reflected and transmitted systems are complementary i.e., a film which appears bright by reflected light will appear dark by transmitted light and vice-versa.

DIFFRACTION OF LIGHT

The phenomenon of bending of light around the corners of an obstacle or aperture is called diffraction of light.

- Diffraction of light is not easily noticed because the obstacles and apertures of size of wavelength of light 10^{-6} m are hardly available.
- In ray optics, we ignore diffraction and assume that light travels in straight lines. This assumption is reasonable because under ordinary conditions, diffraction (bending) of light is negligible.
- The smaller the size of the obstacle or aperture, the greater is the bending (or diffraction) of light around the corners of the obstacle or aperture and vice-versa.

Types of Diffraction

The diffraction phenomenon is generally divided into the following two classes :

- Fraunhofer's diffraction
- Fresnel diffraction

Fresnel diffraction : In this case, either the source or the screen or both are at finite distances from the aperture or obstacle causing diffraction.

Fraunhofer diffraction : In this case, the source and the screen on which the pattern is observed are at infinite distances from the aperture or the obstacle causing diffraction.

Diffraction due to a Single Slit

The diffraction pattern produced by a single slit of width a consists of a central maximum bright band with alternating bright and dark bands of decreasing intensity on both sides of the central maximum.

Condition for n^{th} secondary maximum is,

$$\text{Path difference} = a \sin \theta_n = (2n + 1) \frac{\lambda}{2}$$

where $n = 1, 2, 3, \dots$

Condition for n^{th} secondary minimum is,

$$\text{Path difference} = a \sin \theta_n = n\lambda$$

where $n = 1, 2, 3, \dots$

Width of secondary maxima or minima

$$\beta = \frac{\lambda D}{a} = \frac{\lambda f}{a}$$

where

a = width of slit

D = distance of screen from the slit

f = focal length of lens for diffracted light

$$\text{Width of central maximum} = \frac{2\lambda D}{a} = \frac{2f\lambda}{a}$$

The width of central medium is also called primary fringe width.

$$\text{Angular fringe width of central maximum} = \frac{2\lambda}{a}$$

$$\text{Angular fringe width of secondary maxima or minima} = \frac{\lambda}{a}$$

Fresnel distance

It is the minimum distance a beam of light has to travel before its deviation from straight line path becomes significant. It is given by

$$\text{Fresnel distance, } Z_F = \frac{a^2}{\lambda}$$

Resolving Power of a Microscope

It is defined as the reciprocal of the minimum distance d between two point objects, which can just be seen through the microscope as separate.

$$\text{Resolving power} = \frac{1}{d} = \frac{2\mu \sin \theta}{\lambda}$$

where μ is refractive index of the medium between object and objective lens, θ is half the angle of cone of light from the point object, d represents limit of resolution of microscope and $\mu \sin \theta$ is called the numerical aperture.

Resolving Power of a Telescope

It is defined as reciprocal of the smallest angular separation ($d\theta$) between two distant objects, whose images are just seen in the telescope as separate.

$$\text{Resolving power} = \frac{1}{d\theta} = \frac{D}{1.22 \lambda}$$

where D is diameter or aperture of the objective lens of the telescope, $d\theta$ represents limit of resolution of telescope.

POLARISATION OF LIGHT

The phenomenon of restricting the vibrations of light (electric vector) in a particular direction, perpendicular to direction of wave motion is called polarisation of light.

Polarisation of light confirms the transverse nature of light.

The plane in which vibrations of polarised light are confined is called plane of vibration.

A plane which is perpendicular to the plane of vibration is called plane of polarisation.

Plane polarised light can be produced by the following methods :

- By reflection
- By scattering
- By refraction
- By dichroism
- By double refraction

Angle of Polarisation

The angle of incidence for which an ordinary light is completely polarised in the plane of incidence when it gets reflected from a transparent medium.

Brewster's Law

According to this law, when unpolarised light is incident at polarizing angle i_p , on an interface

separating air from a medium of refractive index μ , then the reflected light is fully polarized (perpendicular to the plane of incidence) provided

$$\mu = \tan i_p.$$

This relation is called Brewster's law.

Note : When the light is incident at polarising angle, the reflected and refracted rays are perpendicular to each other.

Laws of Malus

According to this law when a beam of completely plane polarised light is incident on an analyser, the resultant intensity of light (I) transmitted from the analyser varies directly as the square of the cosine of the angle (θ) between plane of transmission of analyser and polariser *i.e.* $I \propto \cos^2 \theta$

If intensity of plane polarised light incident on analyser is I_0 , then intensity of light emerging from analyser is

$$I = I_0 \cos^2 \theta$$

Note : We can prove that when unpolarised light of intensity I_0 gets polarised on passing through a polaroid, its intensity becomes half,

$$i.e. \quad I = \frac{1}{2} I_0$$

Polaroids

Polaroids can be used to control the intensity, in sunglasses, windowpanes, etc. Polaroids are also used in photographic cameras and 3D movie cameras.

Illustration 11 : A parallel beam of light of wavelength 600 nm is incident normally on a slit of width d . If the distance between the slits and the screen is 0.8 m and the distance of 2nd order maximum from the centre of the screen is 15 mm, find the width of the slit.

Soln.: Distance of 2nd order maximum from the centre of the screen,

$$x'_2 = \frac{5}{2} \frac{D\lambda}{d} \quad \text{or} \quad d = \frac{5}{2} \frac{D\lambda}{x'_2}$$

Given $\lambda = 600 \text{ nm} = 6 \times 10^{-7} \text{ m}$, $D = 0.85 \text{ m}$,

$x'_2 = 15 \text{ mm} = 15 \times 10^{-3} \text{ m}$

$$\therefore d = \frac{5 \times 0.8 \times 6 \times 10^{-7}}{2 \times 15 \times 10^{-3}} = 8 \times 10^{-5} \text{ m} = 80 \mu\text{m}$$

Illustration 12 : Assume that light of wavelength 6000 Å is coming from a star. What is the limit of resolution of telescope whose objective has a diameter of 100 inch?

Soln.: The limit of resolution of a telescope,

$$d\theta = \frac{1.22\lambda}{D}$$

Here $D = 100 \text{ inch} = 254 \text{ cm}$,

$\lambda = 6000 \text{ Å} = 6 \times 10^{-5} \text{ cm}$

$$\therefore d\theta = \frac{1.22 \times 6 \times 10^{-5}}{254} = 2.9 \times 10^{-7} \text{ rad}$$

WAVE PARTICLE DUALITY

Despite their wave nature, electromagnetic radiations, have properties alike to those of particles. Electromagnetic radiation is an emission with a dual nature *i.e.* it has both wave and particle aspects. In particular, the energy conveyed by an electromagnetic wave is always carried in packets whose magnitude is proportional to frequency of the wave. These packets of energy are called photons.

Energy of photon is $E = h\nu$ where h is Planck's constant, and ν is frequency of wave.

According to de Broglie,

- As wave behaves like material particles, similarly matter also behaves like waves. According to him, a wavelength of the matter wave associated with a particle is given by $\lambda = \frac{h}{p} = \frac{h}{mv}$, where m is the mass and v is velocity of the particle.

- If an electron is accelerated through a potential difference of V volt,

$$\text{then } \frac{1}{2} m_e v^2 = eV \quad \text{or} \quad v = \sqrt{\frac{2eV}{m_e}}$$

$$\therefore \lambda = \frac{h}{m_e v} = \frac{h}{\sqrt{2eVm_e}}$$

(It is assumed that the voltage V is not more than several tens of kilovolt)

Illustration 13 : Sun gives light at the rate of 1400 W m^{-2} of area perpendicular to the direction of light. Assume λ (sunlight) = 6000 Å. Calculate the

- number of photons/s arriving at 1 m^2 area at that part of the earth, and
- number of photons emitted from the sun/s assuming the average radius of earth's orbit is $1.49 \times 10^{11} \text{ m}$.

Soln.: $I = 1400 \text{ W m}^{-2}$; $\lambda = 6000 \text{ Å}$, ($c = 3 \times 10^8 \text{ m s}^{-1}$)

- Energy of the photon, $E = h\nu = \frac{hc}{\lambda}$

Let n be the number of photons received/s per unit area.

$$n = \frac{IA}{E/\text{Photon}} = \frac{(1400 \times 1) \times (6000 \times 10^{-10})}{6.63 \times 10^{-34} \times 3 \times 10^8} = 4.22 \times 10^{21}.$$

- Total energy emitted per second = power (watt)

$$n/\text{sec} = \frac{\text{Power of sun (W)}}{E/\text{photon}}$$

$$= \frac{I \times (4\pi R^2) \times (6000 \times 10^{-10})}{6.63 \times 10^{-34} \times 3 \times 10^8}$$

(R average radius of earth's orbit)

$$= 1.178 \times 10^{45}$$

PHOTO ELECTRIC EFFECT

The phenomenon of emission of electrons from a metallic surface when radiation of suitable frequency falls on it is called photo electric effect.

The photo (light) generated electrons are called photo electrons.

Photoelectric effect is a general phenomenon exhibited by all substances but is most easily observed with metals. When radiation of suitable frequency (called threshold frequency) is incident on a metallic surface, electrons are emitted from the metal surface. The threshold frequency is different for different metals.

Some Points :

- If the frequency of incident radiation is equal to or greater than threshold frequency for the metals, electrons will be emitted from the metal, no matter how low is the intensity of radiation.
- If the frequency of incident radiation is less than the threshold frequency for the metal, no photoelectrons will be emitted from the metal surface, no matter how great is the intensity of radiation.

Work function

The minimum amount of work or energy necessary to take a free electron out of a metal against the attractive forces of surrounding positive ions inside metals is called the work function of the metal.

$W_0 = h\nu_0$, where ν_0 is the threshold frequency.

An electron can undergo collisions with other electrons, protons or macroscopically with the atom. In this process it will fritter away its energy. Therefore, electrons with K.E. ranging from 0 to K. E._{max} will be produced.

Einstein's photoelectric equation

According to Einstein, photon energy is utilized for two purposes.

Partly for getting the electron free from the atom and away from the metal surface. This energy is known as the photoelectric work function of the metal and is represented by W_0 .

The balance of the photon energy is used up in giving the electron a kinetic energy of $\frac{1}{2}mv^2$.

$$h\nu = W_0 + \frac{1}{2}mv^2$$

In the case the photon energy is just sufficient to liberate the electron only, the kinetic energy of the electron is zero.

$$\text{i.e., } h\nu_0 = W_0.$$

where ν_0 is the threshold frequency and W_0 is the work function. If the frequency of incident light is less than ν_0 , no photoelectric emission takes place.

Kinetic energy of photoelectrons is

$$\Delta KE = h\nu - h\nu_0 = h(\nu - \nu_0)$$

$$= hc \left(\frac{1}{\lambda} - \frac{1}{\lambda_0} \right) = 12400 \left(\frac{1}{\lambda} - \frac{1}{\lambda_0} \right) \text{ eV}.$$

Illustration 14 : A beam of light has three wavelengths 4144 Å, 4972 Å and 6216 Å with a total intensity of $3.6 \times 10^{-3} \text{ W m}^{-2}$ equally distributed amongst the three wavelengths. The beam falls normally on an area 1.0 cm^2 of a clean metallic surface of work function 2.3 eV. Assuming that there is no loss of light by reflection and that each energetically capable photon ejects one electron. Calculate the number of photo-electrons liberated in 2 seconds.

Soln.: Three different wavelengths are incident on metal surface, so first determine which is (are) capable of ejecting photo-electrons.

For photo-emission, $\lambda \leq \lambda_0$. Given: $W_0 = 2.3 \text{ eV}$

$$W_0 = hc/\lambda_0$$

$$\Rightarrow \lambda_0 = \frac{hc}{W_0} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{2.3 \times 1.6 \times 10^{-19}} = 5404 \text{ Å}$$

\Rightarrow only wavelengths 4144 Å and 4972 Å will cause photo-emission ($6216 \text{ Å} > \lambda_0$)

Intensity of each incident wavelength

$$= 3.6 \times 10^{-3} / 3 = 1.2 \times 10^{-3} \text{ W m}^{-2}$$

[\because I is distributed equally among three wavelengths]

$$n / \text{second} = \frac{IA}{hc / \lambda}$$

$$n / \text{second} (\lambda = 4144 \text{ Å})$$

$$4 \text{ Å}) = \frac{(1.2 \times 10^{-3}) \times (10^{-4}) \times 4144 \times 10^{-10}}{6.63 \times 10^{-34} \times 3 \times 10^8} = 2.5 \times$$

$$n / \text{second} (\lambda = 4972 \text{ Å})$$

$$= \frac{(1.2 \times 10^{-3}) \times (10^{-4}) \times 4972 \times 10^{-10}}{6.63 \times 10^{-34} \times 3 \times 10^8} = 3 \times 10^{11}$$

$$\Rightarrow \text{total electrons emitted/second} = 5.5 \times 10^{11}$$

$$\Rightarrow \text{total electrons emitted in 2 seconds} = 11 \times 10^{11}$$

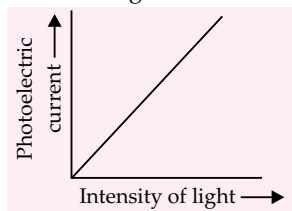
Stopping potential

This is the value of negative potential difference which just stops the electrons with maximum kinetic energy from reaching the anode. If V_s is the stopping potential, then

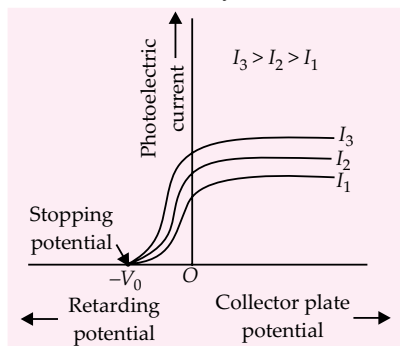
$$eV_s = \frac{1}{2}mv_{\text{max}}^2.$$

Experimental Features and Observations of Photoelectric Effect

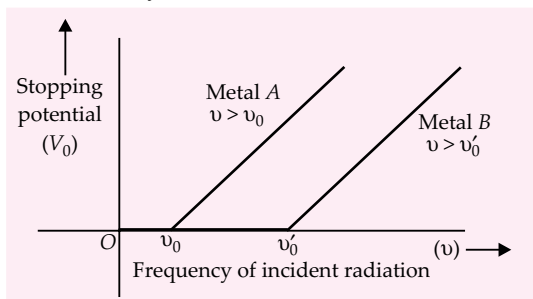
For a given photosensitive material and frequency of incident radiation (above the threshold frequency) the photoelectric current is directly proportional to the intensity of incident light.



- For a given photosensitive material and frequency of incident radiation, saturation current (the maximum value of photoelectric current) is found to be proportional to the intensity of incident radiation whereas the stopping potential is independent of its intensity.



- For a given photosensitive material, there exists a certain minimum cut-off frequency of the incident radiation, called the threshold frequency, below which no emission of photoelectrons takes place, no matter how intense the incident light is. Above the threshold frequency, the stopping potential or equivalently the maximum kinetic energy of the emitted photoelectrons increases linearly with the frequency of incident radiation, but is independent of its intensity.



- The photoelectric emission is an instantaneous process without any apparent time lag ($\sim 10^{-9}$ s or less), even when the incident radiation is made exceedingly dim.

DAVISSON AND GERMER EXPERIMENT

The wave nature of electrons was first experimentally verified by C.J. Davisson and L.H. Germer in 1927 and independently by G.P. Thomson, in 1928, who observed diffraction effects with beams of electrons scattered by crystals.

THOMSON'S MODEL OF ATOM

The first model of atom was proposed by J.J. Thomson in 1898. According to this model, the positive charge of the atom is uniformly distributed throughout the volume of the atom and the negatively charged electrons are embedded in it like seeds in a watermelon. This model was picturesquely called plum pudding model of the atom.

RUTHERFORD'S MODEL OF ATOM

According to this model, the entire positive charge and most of the mass be concentrated in a small region called the nucleus. The electrons revolving around the nucleus in orbits just as the planets revolving around the sun.

Rutherford's Scattering Formula

The formula that Rutherford obtained for alpha particle scattering by a thin foil on the basis of the Rutherford's model of the atom is given by

$$N(\theta) = \frac{N_i n t Z^2 e^4}{(8\pi\epsilon_0)^2 r^2 K^2 \sin^4(\theta/2)}$$

where $N(\theta)$ = number of alpha particles per unit area that reach the screen at a scattering angle of θ
 N_i = total number of alpha particles that reach the screen

n = number of atoms per unit volume in the foil

Z = atomic number of the foil atoms

r = distance of the screen from the foil

K = kinetic energy of the alpha particles

t = foil thickness

The fraction of incident alpha particles scattered by an angle θ or greater is

$$f = \pi n t \left(\frac{Ze^2}{4\pi\epsilon_0 K} \right)^2 \cot^2 \frac{\theta}{2}$$

Impact Parameter

It is defined as the perpendicular distance of the initial velocity vector of the alpha particle from the central line of the nucleus, when the particle is far away from the nucleus of the atom.

The scattering angle θ of the α particle and impact parameter b are related as

$$b = \frac{Ze^2 \cot(\theta/2)}{4\pi\epsilon_0 K}$$

where K is the kinetic energy of the α particle and Z is the atomic number of the nucleus.

Smaller the impact parameter, larger the angle of scattering θ .

Distance of Closest Approach

At the distance of closest approach whole of the kinetic energy of the alpha particles is converted into potential energy.

Distance of closest approach is given by

$$r_0 = \frac{2Ze^2}{4\pi\epsilon_0 K}$$

BOHR'S MODEL OF ATOM

Bohr developed a theory of hydrogen and hydrogen-like atoms which have only one orbital electron. His postulates are as follows :

- An electron can revolve around the nucleus only in certain allowed circular orbits of definite energy and in these orbits it does not radiate. These orbits are called stationary orbits.
- Angular momentum of the electron in a stationary orbit is an integral multiple of $h/2\pi$.

$$\text{i.e., } L = \frac{nh}{2\pi} \quad \text{or, } mvr = \frac{nh}{2\pi}$$

This is called as **Bohr's quantisation condition**.

where m is the mass of the electron, v is the velocity of the electron, r is the radius of the orbit and n is a positive integer called principal quantum number. This postulate is equivalent to saying that in a stationary state, the circumference of a circular orbit contains integral numbers of de Broglie wavelength.

$$2\pi r = n\lambda = \frac{nh}{mv} \quad \text{i.e., } L = mvr = \frac{nh}{2\pi}$$

- The emission of radiation takes place when an electron makes a transition from a higher to a lower orbit. The frequency of the radiation is given by

$$\nu = \frac{E_2 - E_1}{h}$$

where E_2 and E_1 are the energies of the electron in the higher and lower orbits respectively.

Since the centripetal force for circular orbit is provided by the Coulomb's force, we have

$$\frac{1}{4\pi\epsilon_0} \frac{Ze^2}{r^2} = \frac{mv^2}{r}$$

where Z is the atomic number of the element and e is the electronic charge.

Bohr's Formulae

Radius of n^{th} orbit

$$r_n = \frac{4\pi\epsilon_0 n^2 h^2}{4\pi^2 m Z e^2} = \frac{\epsilon_0 n^2 h^2}{\pi m Z e^2}$$

For hydrogen atom, $Z = 1$

$$r_n = \frac{\epsilon_0 n^2 h^2}{\pi m e^2} = a_0 n^2$$

where $a_0 = \frac{h^2 \epsilon_0}{\pi m e^2} = 0.53 \times 10^{-10} \text{ m} = 0.53 \text{ \AA}$ is called

Bohr's radius

Velocity of electron in n^{th} orbit

$$v_n = \frac{1}{4\pi\epsilon_0} \frac{2\pi Z e^2}{n h} = \frac{Z e^2}{2\epsilon_0 n h} = \frac{\alpha c Z}{n} = \frac{c}{137} \frac{Z}{n}$$

where c = speed of light

$$\alpha = \frac{e^2}{2\epsilon_0 h c} = \frac{1}{137}$$

α is called **fine structure constant** and is a pure number.

Frequency of electron in n^{th} orbit

$$\nu_n = \frac{v_n}{2\pi r_n} = \left(\frac{1}{4\pi\epsilon_0} \right)^2 \frac{4\pi^2 Z^2 e^4 m}{n^3 h^3} = \frac{m e^4 Z^2}{4\epsilon_0^2 n^3 h^3}$$

Time period of revolution of electron in n^{th} orbit

$$T_n = \frac{2\pi r_n}{v_n} = \frac{n^3 h^3 (4\pi\epsilon_0)^2}{4\pi^2 Z^2 e^4 m} = \frac{4\epsilon_0^2 n^3 h^3}{m e^4 Z^2}$$

Kinetic energy of electron in n^{th} orbit

$$K_n = \frac{1}{4\pi\epsilon_0} \frac{Z e^2}{2r_n} = \left(\frac{1}{4\pi\epsilon_0} \right)^2 \frac{2\pi^2 m e^4 Z^2}{n^2 h^2} = \frac{13.6 Z^2}{n^2} \text{ eV.}$$

Potential energy of electron in n^{th} orbit

$$U_n = -\frac{1}{4\pi\epsilon_0} \frac{Z e^2}{r_n} = -\left(\frac{1}{4\pi\epsilon_0} \right)^2 \frac{4\pi^2 m e^4 Z^2}{n^2 h^2} = \frac{-27.2 Z^2}{n^2} \text{ eV}$$

Total energy of electron in n^{th} orbit

$$E_n = U_n + K_n = -\left(\frac{1}{4\pi\epsilon_0} \right)^2 \frac{2\pi^2 m e^4 Z^2}{n^2 h^2} = -\frac{13.6 Z^2}{n^2} \text{ eV.}$$

Frequency of emitted radiation

When an electron makes a transition from initial state n_i to final state n_f ($n_i > n_f$) then the frequency of emitted radiation is given by

$$\nu = R c Z^2 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

Wavelength of emitted radiation is given by

$$\frac{1}{\lambda} = RZ^2 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

where R is called **Rydberg's constant**.

$$R = \left(\frac{1}{4\pi\epsilon_0} \right)^2 \frac{2\pi^2 m e^4}{ch^3} = 1.097 \times 10^7 \text{ m}^{-1}.$$

$\frac{1}{\lambda}$ is called **wave number** and is denoted by $\bar{\nu}$.

This relation holds for radiation by hydrogen like atoms *i.e.*

H ($Z = 1$), He⁺ ($Z = 2$), Li⁺⁺ ($Z = 3$) and Be⁺⁺⁺ ($Z = 4$)

Ionization energy and ionization potential

$$\text{Ionization energy} = \frac{13.6Z^2}{n^2} \text{ eV}.$$

$$\text{Ionization potential} = \frac{13.6Z^2}{n^2} \text{ volt}.$$

Hydrogen spectrum

- In emission line spectrum of hydrogen atom, various lines are obtained at different regions of the spectrum. Each group of line is called series of spectral lines and these series are named after the names of their discoverer.

○ The spectral series as shown in the table.

Name of series	Initial state	Final state	Wavelength formula	Wavelength of first line of series (Maximum wavelength)	Series limit (Minimum wavelength)	Region
Lyman	$n_i = 2, 3, 4, 5, \dots$	$n_f = 1$	$\frac{1}{\lambda} = R \left[\frac{1}{1^2} - \frac{1}{n_i^2} \right]$	$\frac{1}{\lambda_{\max}} = R \left[\frac{1}{1^2} - \frac{1}{2^2} \right]$ $\lambda_{\max} = \frac{4}{3R}$	$\frac{1}{\lambda_{\min}} = R \left[\frac{1}{1^2} - \frac{1}{\infty^2} \right]$ $\lambda_{\min} = \frac{1}{R}$	UV
Balmer	$n_i = 3, 4, 5, 6, 7, 8, \dots$	$n_f = 2$	$\frac{1}{\lambda} = R \left[\frac{1}{2^2} - \frac{1}{n_i^2} \right]$	$\frac{1}{\lambda_{\max}} = R \left[\frac{1}{2^2} - \frac{1}{3^2} \right]$ $\lambda_{\max} = \frac{36}{5R}$	$\frac{1}{\lambda_{\min}} = R \left[\frac{1}{2^2} - \frac{1}{\infty^2} \right]$ $\lambda_{\min} = \frac{4}{R}$	Visible
Paschen	$n_i = 4, 5, 6, 7, 8, 9, \dots$	$n_f = 3$	$\frac{1}{\lambda} = R \left[\frac{1}{3^2} - \frac{1}{n_i^2} \right]$	$\frac{1}{\lambda_{\max}} = R \left[\frac{1}{3^2} - \frac{1}{4^2} \right]$ $\lambda_{\max} = \frac{144}{7R}$	$\frac{1}{\lambda_{\min}} = R \left[\frac{1}{3^2} - \frac{1}{\infty^2} \right]$ $\lambda_{\min} = \frac{9}{R}$	Infrared
Brackett	$n_i = 5, 6, 7, 8, \dots$	$n_f = 4$	$\frac{1}{\lambda} = R \left[\frac{1}{4^2} - \frac{1}{n_i^2} \right]$	$\frac{1}{\lambda_{\max}} = R \left[\frac{1}{4^2} - \frac{1}{5^2} \right]$ $\lambda_{\max} = \frac{400}{9R}$	$\frac{1}{\lambda_{\min}} = R \left[\frac{1}{4^2} - \frac{1}{\infty^2} \right]$ $\lambda_{\min} = \frac{16}{R}$	Infrared
Pfund	$n_i = 6, 7, 8, 9, 10, \dots$	$n_f = 5$	$\frac{1}{\lambda} = R \left[\frac{1}{5^2} - \frac{1}{n_i^2} \right]$	$\frac{1}{\lambda_{\max}} = R \left[\frac{1}{5^2} - \frac{1}{6^2} \right]$ $\lambda_{\max} = \frac{900}{11R}$	$\frac{1}{\lambda_{\min}} = R \left[\frac{1}{5^2} - \frac{1}{\infty^2} \right]$ $\lambda_{\min} = \frac{25}{R}$	Far Infrared

Number of spectral lines due to transition of electron from n^{th} orbit to lower orbit is

$$N = \frac{n(n-1)}{2}$$

ENERGY QUANTISATION AND ATOMIC MASSES

In quantum mechanics, the energies of a system are discrete or quantised. The energy of a particle of mass m is confined to a box of length L can have only discrete values of energy given by the relation

$$E_n = \frac{n^2 h^2}{8mL^2} \text{ where } n = 1, 2, 3, \dots$$

Atomic masses refer to the masses of neutral atoms, not of bare nuclei. Thus an atomic mass always includes the masses of its Z electrons. Atomic masses are expressed in atomic mass unit (u) or amu. The value of an atomic mass unit is

$$1 \text{ u} = 1.66054 \times 10^{-27} \text{ kg}$$

The energy equivalent of an atomic mass unit is 931.49 MeV.

NUCLEUS

It exists at the centre of an atom, containing entire positive charge and almost the whole of the mass. The electrons revolve around the nucleus to form an atom. The nucleus consists of protons (+ve charge) and neutrons (no charge).

A proton has positive charge, equal in magnitude to that of an electron, ($1.6 \times 10^{-19} \text{ C}$) and a mass equal to 1836 times that of an electron.

A neutron has no charge and its mass is approximately equal to that of the proton ($1.6726 \times 10^{-27} \text{ kg}$) (1837 times that of an electron).

A particular set of nucleons forming an atom is called a nuclide. It is represented as ${}_Z X^A$.

Isotopes

The nuclides having same number of protons (Z), but different number of nucleons (A) are called **isotopes**.

Isobars

The nuclides having the same number of nucleons (A), but different number of protons (Z) are called **isobars**.

Isotones

The nuclides having the same number of neutrons ($A - Z$) are called **isotones**.

MASS DEFECT AND BINDING ENERGY

The nucleons are bound together in a nucleus and the energy has to be supplied in order to break apart the constituents into free nucleons. The energy with which nucleons are bound together in a nucleus is called **binding energy** (B.E.). In order to free nucleons from a bound nucleus, this much of energy ($= \text{B.E.}$) has to be supplied.

It is observed that the mass of a nucleus is always less than the mass of its constituent (free) nucleons. This difference in mass is called as **mass defect** and is denoted as Δm .

If m_n = mass of neutron and m_p = mass of a proton

$$M(Z, A) = \text{mass of bound nucleus}$$

$$\text{Then, } \Delta m = Zm_p + (A - Z)m_n - M(Z, A)$$

This mass defect is in form of energy and is responsible for binding the nucleons together. From Einstein's mass-energy relation,

$$E = mc^2 \text{ (} c \text{ is speed of light, } m \text{ is mass)}$$

$$\Rightarrow \text{Binding energy} = \Delta mc^2$$

Generally, Δm is measured in amu units. So let us calculate the energy equivalent to 1 amu. It is calculated in eV (electron volt, $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$)

$$E (\equiv 1 \text{ amu}) = \frac{1 \times 1.67 \times 10^{-27} \times (3 \times 10^8)^2}{1.6 \times 10^{-19}} \text{ eV}$$

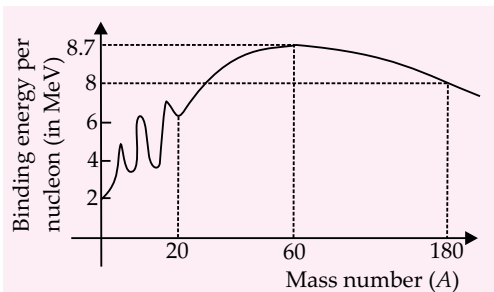
$$= 931 \times 10^6 \text{ eV} = 931 \text{ MeV}$$

$$\Rightarrow \text{B.E.} = \Delta m(931) \text{ MeV}$$

There is another quantity which is very useful in predicting the stability of a nucleus called as binding energy per nucleon.

$$\text{B.E. per nucleon} = \frac{\Delta m(931)}{A} \text{ MeV.}$$

From the Plot of B.E./Nucleon Vs Mass Number (A), we observe that :



- B.E./nucleon increases on an average and reaches a maximum of about 8.7 MeV for $A \approx 50-80$.
- For heavier nuclei, B.E./nucleon decreases slowly as A increases. For the heaviest natural element ${}^{238}\text{U}$ it drops to about 7.5 MeV.
- From above observation, it follows that nuclei in the region of atomic masses 50-80 are most stable.

NUCLEAR FORCES

The protons and neutrons are held together by the strong attractive forces inside the nucleus. These forces are called as nuclear forces.

Properties of the Nuclear Force

- Nuclear force is short ranged. It exists in small region (of diameter $10^{-15} \text{ m} = 1 \text{ fm}$). The nuclear force between two nucleons decreases rapidly as the separation between them increases and becomes negligible at separation more than 10 fm.

- Nuclear force is much stronger than electromagnetic force and gravitational force.
- Nuclear force is independent of charge. The nuclear force between two protons is same as that between two neutrons or between a neutron and proton. This is known as charge independent character of nuclear force.

NUCLEAR REACTION

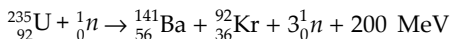
In nuclear reaction, sum of masses before reaction is greater than the sum of masses after the reaction. The difference in masses appears in the form of energy following the law of inter-conversion of mass and energy. The energy released in a nuclear reaction is called as Q value of a reaction and is given as follows :
If difference in mass before and after the reaction is Δm amu

Δm = mass of reactants minus mass of products, then
 Q value = $\Delta m(931)$ MeV

Law of conservation of momentum is also followed. Total number of protons and neutrons should also remain same on both sides of a nuclear reaction.

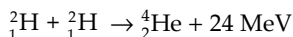
Nuclear Fission

The breaking of a heavy nucleus into two or more fragments of comparable masses, with the release of tremendous amount of energy is called as nuclear fission. The most typical fission reaction occurs when slow moving neutrons strike ${}_{92}^{235}\text{U}$. The following nuclear reaction takes place.



Nuclear Fusion

The process in which two or more light nuclei are combined into a single nucleus with the release of tremendous amount of energy is called as nuclear fusion. Like a fission reaction, the sum of masses before the fusion (*i.e.* of light nuclei) is more than the sum of masses after the fusion (*i.e.* of bigger nucleus) and this difference appears as the fusion energy. The most typical fusion reaction is the fusion of two deuterium nuclei into helium.



For the fusion reaction to occur, the light nuclei are brought closer to each other (with a distance of 10^{-14} m). This is possible only at very high temperature to counter the repulsive force between nuclei. Due to this reason, the fusion reaction is very difficult to perform. The inner core of sun is at very high temperature, and is suitable for fusion, in fact the source of energy in sun and other stars is the nuclear fusion reaction.

Illustration 15 : It is proposed to use the nuclear fusion reaction, ${}_1^2\text{H} + {}_1^2\text{H} \rightarrow {}_2^4\text{He}$ in a nuclear reactor, of 200 MW rating. If the energy from above reaction is used with a 25% efficiency in the reactor, how many grams of deuterium will be needed per day? (The masses of ${}_1^2\text{H}$ and ${}_2^4\text{He}$ are 2.0141 amu and 4.0026 amu respectively.)

Soln.: Let us first calculate the Q value of nuclear reaction.

$$\begin{aligned} Q &= \Delta mc^2 = \Delta m(931) \text{ MeV} \\ \Rightarrow Q &= (2 \times 2.0141 - 4.0026) \times 931 \text{ MeV} \\ &= 23.834 \text{ MeV} = 23.834 \times 10^6 \text{ eV.} \end{aligned}$$

Now efficiency of reactor is 25%.

So effective energy used

$$= \frac{25}{100} \times 23.834 \times 10^6 \times 1.6 \times 10^{-19} \text{ J} = 9.534 \times 10^{-13} \text{ J}$$

Now 9.534×10^{-13} J energy is released by fusion of 2 deuterium.

$$\Rightarrow \frac{(9.534 \times 10^{-13})}{2} \text{ J/deuterium is released.}$$

Requirement is 200 MW = $200 \times 10^6 \text{ J s}^{-1} \times 86400$ for 1 day.

No. of deuterium nuclei required

$$= \frac{200 \times 10^6 \times 86400}{\frac{9.534}{2} \times 10^{-13}} = 3.624 \times 10^{25}$$

$$\text{Number of deuterium nuclei} = \frac{m}{M} \times 6 \times 10^{23}$$

$$3.624 \times 10^{25} = \frac{m}{2} \times 6 \times 10^{23}$$

$$\Rightarrow m = \frac{2 \times 3.624 \times 10^{25}}{6 \times 10^{23}} = 120.83 \text{ g day}^{-1}$$

RADIOACTIVITY

The phenomenon of spontaneous emission of radiation or particles from the nucleus is called radioactivity. The substances which emit these radiations are called as radioactive substances. It was discovered by Henry Becquerel for atoms of radium. Later it was discovered that many naturally occurring compounds of heavy elements like radium, thorium etc also emit radiations.

At present, it is known that all the naturally occurring elements having atomic number greater than 82 are radioactive. For example some of them are; radium, polonium, thorium, actinium, uranium, radon etc. Later on Rutherford found that emission of radiation always accompanied by transformation of one element (transmutation) into another. Actually radioactivity is the result of disintegration of an unstable nucleus. Rutherford studied the nature of these radiations and found that these mainly consist of α , β , γ rays.

α -Particles (${}^4_2\text{He}$)

These carry a charge of $+2e$ and mass equal to $4m_p$. These are nuclei of helium atoms. The energies of α -particles vary from 5 MeV to 9 MeV and their velocities vary from 0.01-0.1 times of c (velocity of light). They can be deflected by electric and magnetic fields and have low penetrating power but high ionizing power.

β -Particles (${}^0_{-1}e$)

These are fast moving electrons having charge equal to $-e$ and mass $m_e = 9.1 \times 10^{-31}$ kg. Their velocities vary from 1% to 99% of the velocity of light (c). They can also be deflected by electric and magnetic fields. They have low ionizing power but high penetrating power. β^+ particles are positrons.

γ -Radiation (${}^0_0\gamma$)

These are electromagnetic waves of nuclear origin and of very short wavelength. They have no charge and no mass. They have maximum penetrating power and minimum ionising power. The energy released in a nuclear reaction is mainly emitted in the form of γ radiation.

Laws of Radioactive Decay

Rutherford-Soddy laws (Statistical Laws)

- The disintegration of a radioactive substance is random and spontaneous.
- Radioactive decay is purely a nuclear phenomenon and is independent of any physical and chemical conditions.
- The radioactive decay follows first order kinetics, i.e., the rate of decay is proportional to the number of undecayed atoms in a radioactive substance at any time t .
- If dN be the number of atoms (nuclei) disintegrating in time dt , the rate of decay is given as dN/dt . From first order of kinetic rate law

$\frac{dN}{dt} = -\lambda N$, where λ is called as decay or disintegration constant.

Let N_0 be the number of nuclei at time $t = 0$ and N_t be the number of nuclei after time t , then according to integrated first order rate law, we have

$$N_t = N_0 e^{-\lambda t} \Rightarrow \lambda t = \ln \frac{N_0}{N_t} = 2.303 \log \frac{N_0}{N_t}$$

The **half life** ($T_{1/2}$) period of a radioactive substance is defined as the time in which one-half of the radioactive substance is disintegrated. If N_0 be the number of nuclei at $t = 0$, then in a half life $T_{1/2}$, the number of nuclei decayed will be $N_0/2$.

$$N_t = N_0 e^{-\lambda t} \quad \dots(i)$$

$$\Rightarrow \frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}} \quad \dots(ii)$$

From (i) and (ii), we get

$$\frac{N_t}{N_0} = \left(\frac{1}{2}\right)^{t/T_{1/2}} = \left(\frac{1}{2}\right)^n$$

n = number of half lives

The **mean life** (T_m) of a radioactive substance is equal to the sum of life times of all atoms divided by the number of all atoms. It is given by

$$T_m = \frac{1}{\lambda}$$

Illustration 16 : The mean lives of a radio active substance are 1620 and 405 years for α -emission and β -emission respectively. Find out the time during which three fourth of a sample will decay if it is decaying both the α -emission and β -emission simultaneously.

Soln.: When a substance decays by α and β emission simultaneously, the average rate of disintegration λ_{av} is given by

$$\lambda_{av} = \lambda_{\alpha} + \lambda_{\beta}$$

where λ_{α} = disintegration constant for α -emission only

λ_{β} = disintegration constant for β -emission only.

Mean life is given by

$$T_m = \frac{1}{\lambda}$$

$$\begin{aligned} \Rightarrow \lambda_{av} &= \lambda_{\alpha} + \lambda_{\beta} \Rightarrow \frac{1}{T_m} = \frac{1}{T_{\alpha}} + \frac{1}{T_{\beta}} \\ &= \frac{1}{1620} + \frac{1}{405} = 3.08 \times 10^{-3} \end{aligned}$$

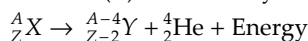
$$\lambda_{av} t = 2.303 \log \frac{100}{25}$$

$$(3.08 \times 10^{-3}) t = 2.303 \log \frac{100}{25}$$

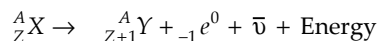
$$\Rightarrow t = 2.303 \times \frac{1}{3.08 \times 10^{-3}} \log 4 = 450.17 \text{ years.}$$

Soddy Fajan Laws (Group-Displacement Laws)

- When a nuclide emits one α -particle (${}^4_2\text{He}$), its mass number (A) decreases by 4 units and atomic number (Z) decreases by 2 units.



- When a nuclide emits a β -particle, its mass number remains unchanged but atomic number increases by one unit.



where $\bar{\nu}$ is antineutrino.

In the nucleus, due to conversion of neutron into proton, antineutrino is produced. It has no charge

Wave Motion

Progressive Waves

- For a plane progressive harmonic wave travelling along -ve X-axis, Displacement, $y = A \sin(\omega t + kx)$

$$y = A \sin 2\pi \left(\frac{t}{T} + \frac{x}{\lambda} \right) = A \sin \frac{2\pi}{\lambda} (vt + x)$$
- Phase, $\phi = 2\pi \left(\frac{t}{T} + \frac{x}{\lambda} \right) + \phi_0$
 where ϕ_0 is the initial phase.
- Phase change with time,

$$\Delta\phi = \frac{2\pi}{T} \Delta t.$$
- Phase change with position,

$$\Delta\phi = \frac{2\pi}{\lambda} \Delta x.$$
- Instantaneous particle velocity,

$$u = \frac{dy}{dt} = \frac{2\pi A}{T} \cos 2\pi \left(\frac{t}{T} + \frac{x}{\lambda} \right)$$
- Velocity amplitude,

$$u_0 = \frac{2\pi A}{T} = \omega A$$
- Instantaneous particle acceleration,

$$a = \frac{du}{dt} = -\frac{4\pi^2}{T^2} A \sin 2\pi \left(\frac{t}{T} + \frac{x}{\lambda} \right)$$

$$= -\omega^2 y$$
- Acceleration amplitude,

$$a_0 = \frac{4\pi^2}{T^2} A = \omega^2 A$$
- Average power transmitted along the string by a sine wave

$$P_{av} = \frac{1}{2} \frac{\omega^2 A^2 T}{v} = 2\pi^2 m v A^2 \nu^2$$

Stationary Waves

- The stationary wave formed by the superposition of incident wave and reflected wave is given by

$$y = \pm 2a \sin \frac{2\pi x}{\lambda} \cos \frac{2\pi t}{T}$$
- For (+) sign in the equation, antinodes are formed at the positions

$$x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \dots$$

 and nodes are formed at

$$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \dots$$
- For (-) sign, antinodes are formed at the positions,

$$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \dots$$

 and nodes at $x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \dots$
- The distance between two successive nodes or antinodes is $\frac{\lambda}{2}$ and that between a node and nearest antinode is $\frac{\lambda}{4}$.

Doppler's Effect in Sound

- If v , v_o , v_s and v_m are the velocities of sound, observer, source and medium respectively, then the apparent frequency,

$$\nu' = \frac{v + v_m - v_o}{v + v_m - v_s} \times \nu$$
- If the medium is at rest, ($v_m = 0$) then

$$\nu' = \frac{v - v_o}{v - v_s} \times \nu$$
- All the velocities are taken positive in the source to observer direction and negative in the observer to source direction.

Relation between Frequency, Wavelength and Wave velocity

$$v = \nu \lambda = \frac{1}{T} \lambda$$

Velocity of Transverse Wave in Solids and Strings

- In solids, $v = \sqrt{\frac{\eta}{\rho}}$
 where η is modulus of rigidity and ρ is density of solids.
- In stretched string, $v = \sqrt{\frac{T}{m}}$
 here, T is tension in string and m is mass per unit length of string.

Velocity of Longitudinal Waves

- Velocity of longitudinal waves in a solid of bulk modulus κ , modulus of rigidity η and density ρ is

$$v = \sqrt{\frac{\kappa + \frac{4}{3}\eta}{\rho}}$$

- Velocity of longitudinal waves in a long solid rod of Young's modulus Y and density ρ is given by

$$v = \sqrt{\frac{Y}{\rho}}$$

- Velocity of longitudinal waves in a fluid of bulk modulus κ and density ρ is

$$v = \sqrt{\frac{\kappa}{\rho}}$$

- Newton's formula for the velocity of sound in a gas is

$$v = \sqrt{\frac{\kappa_{iso}}{\rho}} = \sqrt{\frac{P}{\rho}}$$

here, P = pressure of the gas

- Laplace formula for the velocity of sound in a gas is

$$v = \sqrt{\frac{\kappa_{adia}}{\rho}} = \sqrt{\frac{\gamma P}{\rho}}$$

here, $\gamma = \frac{C_p}{C_v}$

- Intensity of sound waves

$$I = \frac{1}{2} \frac{\omega^2 A^2 \kappa}{v} = \frac{2\pi^2 \kappa}{v} A^2 \nu^2 = \frac{P_0^2 \nu}{2\kappa} = \frac{P_0^2}{2\rho v}$$

Factors affecting velocity of Sound through gases

- Effect of density, $v \propto \frac{1}{\sqrt{\rho}}$
 i.e., $\frac{v_2}{v_1} = \sqrt{\frac{\rho_1}{\rho_2}}$
- Effect of temperature, $v \propto \sqrt{T}$

$$\frac{v_t}{v_0} = \sqrt{\frac{T}{T_0}} = \sqrt{\frac{273+t}{273}}$$
- Effect of pressure : No change in velocity of sound with change in pressure.

Modes of Vibration of strings

- Frequency of vibration of a string fixed at both ends

$$\nu = \frac{nv}{2L} = \frac{n}{2L} \sqrt{\frac{T}{m}}$$

where L = length of string
 n = mode of vibration

- Fundamental frequency

$$\nu_0 = \frac{v}{2L} = \frac{1}{2L} \sqrt{\frac{T}{m}}$$

is the lowest frequency of vibration producing fundamental note or first harmonic.

- Frequency of vibration of a string fixed at one end.

$$\nu = \left(n + \frac{1}{2} \right) \frac{v}{2L} = \left(\frac{n+1}{2} \right) \sqrt{\frac{T}{m}}$$

- Fundamental frequency,

$$\nu_0 = \frac{v}{4L} = \frac{1}{4L} \sqrt{\frac{T}{m}}$$

- Second harmonic or 1st overtone, $\nu_2 = 2\nu_0$
- Third harmonic or 2nd overtone, $\nu_3 = 3\nu_0$ and so on.
- Law of length
 $\nu L = \text{constant}$
 or $\nu_1 L_1 = \nu_2 L_2$

Organ Pipes

- Open organ pipe
 Fundamental mode,

$$\nu_1 = \frac{v}{2L} = \nu \quad (\text{First harmonic})$$

Second mode, $\nu_2 = 2\nu$
 (Second harmonic or first overtone)

n^{th} mode, $\nu_n = \frac{n\nu}{2L}$
 (n^{th} harmonic or $(n-1)^{\text{th}}$ overtone)

- Closed organ pipe
 Fundamental mode,

$$\nu_1 = \frac{v}{4L} = \nu \quad (\text{First harmonic})$$

Second mode, $\nu_2 = 3\nu$
 (Third harmonic or first overtone)

Third mode, $\nu_3 = 5\nu$
 (Fifth harmonic or second overtone)

n^{th} mode, $\nu_n = (2n-1)\nu$
 $[(2n-1)^{\text{th}}$ harmonic or $(n-1)^{\text{th}}$ overtone]

- Resonance tube
 If L_1 and L_2 are the first and second resonance lengths with a tuning fork of frequency ν , then speed of sound

$$v = 4\nu (L_1 + 0.3 D),$$

D = internal diameter of resonance tube

or $v = 2\nu (L_2 - L_1)$
 End correction = $0.3 D = \frac{L_2 - 3L_1}{2}$

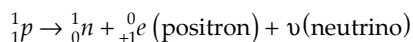
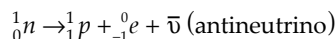
Beats Formation

- Beat frequency = Number of beats/sec
 = Difference in frequencies of two sources.

$$\nu_{\text{beat}} = (\nu_1 - \nu_2) \text{ or } (\nu_2 - \nu_1)$$

$$\therefore \nu_2 = \nu_1 + \nu_{\text{beat}}$$
- The \pm sign is decided by loading/filing any of the prongs of either tuning fork.
- On loading a fork, its frequency decreases and on filing, its frequency increases.

or mass, but has momentum. When a proton is converted to a neutron, a neutron and a $+\text{ve } \beta$ -particle is produced, which is called as positron. β rays are electrons and β^+ are the antielectrons or positrons.



Antineutrino and neutrino share the energy of electrons and positrons. That is the reason why the energy of β is continuous and β rays has a energy maximum.

- When a γ particle is produced, both atomic and mass number remain constant.

Activity of a Radioactive Isotope

The activity of a radioactive substance (or radioisotope) means the rate of decay per second or the number of nuclei disintegrating per second. It is generally denoted by A .

$$A = \frac{dN}{dt}$$

At time $t = 0$, the activity of a radioactive substance be A_0 and after time $t = t$ s, activity be A_t then

$$A_0 = \left[\frac{dN}{dt} \right]_{t=0} = -\lambda N_0$$

$$A_t = \left[\frac{dN}{dt} \right]_{t=t} = -\lambda N_t$$

$$A_t = A_0 e^{-\lambda t}$$

Unit of Activity

The activity is measured in terms of curie (Ci). 1 curie is the activity of 1 g of a freshly prepared sample of radium ${}^{226}\text{Ra}$ ($T_{1/2} = 1602$ years.)

1 curie = 1 Ci = 3.7×10^{10} dps (disintegration per second)
1 dps is also known as 1 Bq (becquerel)

$$\Rightarrow 1 \text{ Ci} = 3.7 \times 10^{10} \text{ Bq}$$

Illustration 17 : Radioisotopes of phosphorus ${}^{32}\text{P}$ and ${}^{35}\text{P}$ are mixed in the ratio of 2 : 1 of atoms. The activity of the sample is 2 Ci. Find the activity of the sample after 30 days. $T_{1/2}$ of ${}^{32}\text{P} = 14$ days and $T_{1/2}$ of ${}^{35}\text{P}$ is 25 days.

Soln.: Let A_0 = initial activity of sample.

A_{10} = initial activity of isotope 1 and

A_{20} = initial activity of isotope 2.

$$A_0 = A_{1t} + A_{2t}$$

Similarly for final activity (Activity after time t)

$$A_t = A_{1t} + A_{2t}$$

$$\Rightarrow A_t = A_{10} e^{-\lambda_1 t} + A_{20} e^{-\lambda_2 t}$$

Now in the given equation,

$$A_0 = 2 \text{ Ci} \Rightarrow A_0 = A_{10} + A_{20} = 2 \quad \dots(i)$$

Initial ratio of atoms of isotopes = 2 : 1

From definition of activity,

$$A = \lambda N$$

$$\Rightarrow \frac{A_{10}}{A_{20}} = \frac{\lambda_1 N_{10}}{\lambda_2 N_{20}} = \frac{N_{10}}{N_{20}} \times \frac{T_2}{T_1}$$

where T represents half life

$$\Rightarrow \frac{A_{10}}{A_{20}} = \frac{2}{1} \times \frac{25}{14} = \frac{50}{14} = \frac{25}{7} \quad \dots(ii)$$

On solving equation (i) and (ii), we get

$$A_{20} = \frac{7}{16} \text{ and } A_{10} = \frac{25}{16}$$

$$A_t = A_{10} e^{-\lambda_1 t} + A_{20} e^{-\lambda_2 t}$$

$$\Rightarrow A_t = \frac{25}{16} e^{-\frac{0.693}{14} \times 30} + \frac{7}{16} e^{-\frac{0.693}{25} \times 30}$$

Consider the first exponential term:

$$e^{-\frac{0.693 \times 30}{14}} = e^{-1.485}$$

$$\text{Let } y = e^{-1.485} \Rightarrow \ln y = -1.485$$

$$\Rightarrow \log y = \frac{-1.485}{2.303} \Rightarrow y = \text{antilog} \left(\frac{-1.485}{2.303} \right)$$

So, from above calculations you can derive a general result i.e.,

$$e^{-x} = \text{antilog} \left(\frac{-x}{2.303} \right)$$

$$A_t = \frac{25}{16} \times 0.2265 + \frac{7}{16} \times 0.4354 = 0.5444 \text{ Ci.}$$

Form IV

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I, Mahabir Singh, here by declare that particulars given above are true to the best of my knowledge and belief.

Mahabir Singh
Publisher

QUESTIONS FOR PRACTICE

1. A convex lens forms an image of an object placed 20 cm away from it at a distance of 20 cm on the other side of the lens. If the object is moved 5 cm towards the lens, the image will move
 - (a) 5 cm towards the lens
 - (b) 5 cm away from the lens
 - (c) 10 cm towards the lens
 - (d) 10 cm away from the lens
2. The Fraunhofer diffraction pattern of a single slit is formed in the focal plane of a lens of focal length 1 m. The width of slit is 0.3 mm. If third minimum is formed at a distance of 5 mm from central maximum, then wavelength of light will be
 - (a) 5000 Å
 - (b) 2500 Å
 - (c) 7500 Å
 - (d) 8500 Å
3. Energy required to remove an electron from an aluminium surface is 4.2 eV. If light of wavelength 2000 Å falls on the surface, the velocity of faster electrons ejected from the surface is
 - (a) $2.5 \times 10^{18} \text{ m s}^{-1}$
 - (b) $2.5 \times 10^{13} \text{ m s}^{-1}$
 - (c) $6.7 \times 10^{18} \text{ m s}^{-1}$
 - (d) $8.4 \times 10^5 \text{ m s}^{-1}$
4. The half-life for the α -decay of Uranium ${}_{92}^{238}\text{U}$ is $4.47 \times 10^9 \text{ yr}$. If a rock contains sixty percent of its original ${}_{92}^{238}\text{U}$ atoms, its age is
 [Given, $\log 6 = 0.778$; $\log 2 = 0.3$]
 - (a) $3.3 \times 10^9 \text{ yr}$
 - (b) $6.6 \times 10^9 \text{ yr}$
 - (c) $1.2 \times 10^8 \text{ yr}$
 - (d) $5.4 \times 10^7 \text{ yr}$
5. A ray of light incident at an angle θ on a refracting face of a prism emerges from the other face normally. If the angle of the prism is 5° and the prism is made of a material of refractive index 1.5, the angle of incidence is
 - (a) 7.5°
 - (b) 5°
 - (c) 15°
 - (d) 2.5°
6. Consider sunlight incident on a slit of width 10^4 Å . The image seen through the slit shall
 - (a) be a fine sharp slit white in colour at the centre
 - (b) a bright slit white at the centre diffusing to zero intensities at the edges
 - (c) a bright slit white at the centre diffusing to regions of different colours
 - (d) only be a diffused slit white in colour
7. Light rays of wavelength 6000 Å and of photon intensity 39.6 W m^{-2} is incident on a metal surface. If only 1% of photons incident on surface emit photoelectrons, then the number of electrons emitted per second per unit area from the surface will be
 (Given, $h = 6.64 \times 10^{-34} \text{ J s}$, $c = 3 \times 10^8 \text{ m s}^{-1}$)
 - (a) 12×10^{18}
 - (b) 10×10^{18}
 - (c) 1.2×10^{18}
 - (d) 12×10^{16}
8. If the atom ${}_{100}^{257}\text{Fm}$ follows the Bohr model and the radius of fifth orbit of ${}_{100}^{257}\text{Fm}$ is N times the Bohr radius, then find the value of N is
 - (a) 100
 - (b) 200
 - (c) 4
 - (d) $1/4$
9. What is the refractive index of material of a planoconvex lens, if the radius of curvature of the convex surface is 10 cm and focal length of the lens is 30 cm?
 - (a) $\frac{6}{5}$
 - (b) $\frac{7}{4}$
 - (c) $\frac{2}{3}$
 - (d) $\frac{4}{3}$
10. In a Young's double slit experiment, the fringes are displaced by a distance x when a glass plate of refractive index 1.5 is introduced in the path of one of the beams. When this plate is replaced by another plate of same thickness, the shift of fringes is $(3/2)x$. The refractive index of second plate is
 - (a) 1.75
 - (b) 1.50
 - (c) 1.25
 - (d) 1.00
11. The binding energy of deuteron (${}_1^2\text{H}$) is 1.15 MeV per nucleon and an alpha particle (${}_2^4\text{He}$) has binding energy of 7.1 MeV per nucleon. Then in the reaction ${}_1^2\text{H} + {}_1^2\text{H} \rightarrow {}_2^4\text{He} + Q$, the energy released Q is
 - (a) 5.95 MeV
 - (b) 26.1 MeV
 - (c) 23.8 MeV
 - (d) 289.4 MeV
12. In Young's experiment the wavelength of red light is $7.5 \times 10^{-5} \text{ cm}$ and that of blue light $5.0 \times 10^{-5} \text{ cm}$. The value of n for which $(n+1)^{\text{th}}$ blue bright band coincides with n^{th} red band is
 - (a) 8
 - (b) 4
 - (c) 2
 - (d) 1
13. The magnifying power of a telescope is 9. When it is adjusted for parallel rays, the distance between the objective and the eye-piece is found to be 20 cm. The focal lengths of the lenses are
 - (a) 18 cm, 2 cm
 - (b) 11 cm, 9 cm
 - (c) 10 cm, 10 cm
 - (d) 15 cm, 5 cm

14. An object is placed 30 cm to the left of a diverging lens whose focal length is of magnitude 20 cm. Which one of the following correctly states the nature and position of the virtual image formed?

Nature of image Distance from lens

- (a) inverted, enlarged 60 cm to the right
 (b) erect, diminished 12 cm to the left
 (c) inverted, enlarged 60 cm to the left
 (d) erect, diminished 12 cm to the right
15. The angular momentum of electron in $3d$ orbital of an atom is
- (a) $\sqrt{2}\left(\frac{h}{2\pi}\right)$ (b) $\sqrt{3}\left(\frac{h}{2\pi}\right)$
 (c) $\sqrt{6}\left(\frac{h}{2\pi}\right)$ (d) $\sqrt{12}\left(\frac{h}{2\pi}\right)$
16. The binding energy of an electron in the ground state of He is equal to 24.6 eV. The energy required to remove both the electrons is
- (a) 49.2 eV (b) 24.6 eV
 (c) 38.2 eV (d) 79.0 eV
17. A convex lens of focal length 0.15 m is made of a material of refractive index $\frac{3}{2}$. When it is placed in a liquid, its focal length is increased by 0.225 m. The refractive index of the liquid is
- (a) $\frac{7}{4}$ (b) $\frac{5}{4}$
 (c) $\frac{9}{4}$ (d) $\frac{3}{2}$
18. A transparent thin plate of a polaroid is placed on another similar plate such that the angle between their axes is 30° . The intensities of the emergent and the unpolarized incident light will be in the ratio of
- (a) 1 : 4 (b) 1 : 3 (c) 3 : 4 (d) 3 : 8
19. When an object is kept at a distance of 30 cm from a concave mirror, the image is formed at a distance of 10 cm. If the object is moved with a speed of 9 m s^{-1} , the speed with which image moves is
- (a) 10 m s^{-1} (b) 1 m s^{-1}
 (c) 9 m s^{-1} (d) 0.9 m s^{-1}
20. When the electromagnetic radiations of frequencies $4 \times 10^{15} \text{ Hz}$ and $6 \times 10^{15} \text{ Hz}$ fall on the same metal, in different experiments, the ratio of maximum kinetic energy of electrons liberated is 1 : 3. The threshold frequency for the metal is
- (a) $2 \times 10^{15} \text{ Hz}$ (b) $1 \times 10^{15} \text{ Hz}$
 (c) $3 \times 10^{15} \text{ Hz}$ (d) $4 \times 10^{15} \text{ Hz}$
21. ${}^7_3\text{Li}$ nucleus has three protons and four neutrons. Mass of ${}^7_3\text{Li}$ nucleus is 7.016005 amu. Mass of

proton is 1.007277 amu and mass of neutron is 1.008665 amu. Mass defect of lithium nucleus in amu is

- (a) 0.04048 (b) 0.04050
 (c) 0.04052 (d) 0.04055

Directions : Question number 22 and 23 have Statement-I and Statement-II. Of the four choices given after the Statements, choose the one that best describes the two Statements.

- (a) Statement-I is false, Statement-II is true.
 (b) Statement-I is true, Statement-II is true, Statement-II is the correct explanation of Statement-I.
 (c) Statement-I is true, Statement-II is true, Statement-II is not the correct explanation of Statement-I.
 (d) Statement-I is true, Statement-II is false.

22. **Statement-I :** The pattern and position of fringes shift after the introduction of a transparent medium in the path of one of the slits.

Statement-II : The central fringe is bright or dark does not depend upon the initial phase difference between the two coherent sources.

23. **Statement-I :** A biconvex lens of focal length 10 cm is split into two equal parts by a plane parallel to its principal axis. The focal length of each part will be 20 cm.

Statement-II : Focal length depends on the radii of curvature of two surfaces.

24. Taking the Bohr radius as $a_0 = 53 \text{ pm}$, the radius of Li^{++} ion in its ground state, on the basis of Bohr's model, will be about
- (a) 53 pm (b) 27 pm (c) 18 pm (d) 13 pm

25. In a nuclear reactor, moderators slow down the neutrons which come out in a fission process. The moderator used have light nuclei. Heavy nuclei will not serve the purpose because

- (a) they will break up
 (b) elastic collision of neutrons with heavy nuclei will not slow them down
 (c) the net weight of the reactor would be unbearably high
 (d) substances with heavy nuclei do not occur in liquid or gaseous state at room temperature.

26. A plastic sheet (refractive index = 1.6) covers one slit of a double slit arrangement meant for the Young's experiment. When the double slit is illuminated by monochromatic light (wavelength in air = 6600 \AA), the centre of the screen appears dark rather than bright. The minimum thickness of the plastic sheet to be used for this to happen is

- (a) 3300 \AA (b) 6600 \AA
 (c) 2062 \AA (d) 5500 \AA

27. Light of wavelength 1500 \AA fall on aluminium surface. Work function of aluminium is 4.2 eV . What is the kinetic energy of the fastest emitted photoelectrons?
(a) 2 eV (b) 1 eV (c) 4 eV (d) 0.2 eV
28. If the series limit wavelength of Lyman series for the hydrogen atom is 912 \AA , then the series limit wavelength for Balmer series of hydrogen atom is
(a) 912 \AA (b) $912 \times 2 \text{ \AA}$
(c) $912 \times 4 \text{ \AA}$ (d) $\frac{912}{2} \text{ \AA}$
29. One of the refracting surfaces of a prism of angle 30° is silvered. A ray of light incident at an angle of 60° retraces its path. The refractive index of the material of prism is
(a) $\sqrt{3}$ (b) $\frac{3}{2}$ (c) 2 (d) $\sqrt{2}$
30. The focal length of objective and eye-piece of a microscope are 1 cm and 5 cm respectively. If the magnifying power for relaxed eye is 45 , then length of the tube is
(a) 9 cm (b) 15 cm (c) 12 cm (d) 6 cm

SOLUTIONS

1. (d): Clearly, $2f = 20 \text{ cm}$ or $f = 10 \text{ cm}$
Now, $u = -15 \text{ cm}$, $f = 10 \text{ cm}$
Using lens formula, $\frac{1}{v} - \frac{1}{-15} = \frac{1}{10}$
or $\frac{1}{v} + \frac{1}{15} = \frac{1}{10}$ or $\frac{1}{v} = \frac{1}{10} - \frac{1}{15}$
or $\frac{1}{v} = \frac{3-2}{30} = \frac{1}{30}$ or $v = 30 \text{ cm}$
The change in image distance is $(30 - 20) \text{ cm}$ i.e., 10 cm away from the lens.
2. (a): As for minima, $n\lambda = a \sin \theta = \frac{ax}{f}$ or $\lambda = \frac{ax}{nf}$
$$\left[\because \sin \theta = \frac{x}{f} \right]$$

Here, $a = 0.3 \text{ mm} = 0.3 \times 10^{-3} \text{ m}$, $x = 5 \text{ mm} = 5 \times 10^{-3} \text{ m}$,
 $n = 3$, $f = 1 \text{ m}$.
 $\therefore \lambda = \frac{0.3 \times 10^{-3} \times 5 \times 10^{-3}}{3 \times 1} = 5 \times 10^{-7} \text{ m} = 5000 \text{ \AA}$.
3. (d): From $\frac{1}{2}mv^2 = \frac{hc}{e\lambda} - \phi_0$ (in eV)
 $\therefore \frac{1}{2}mv^2 = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{2000 \times 10^{-10} \times 1.6 \times 10^{-19}} - 4.2 = 6.2 - 4.2$
 $= 2 \text{ eV} = 2 \times 1.6 \times 10^{-19} \text{ J}$

$$\Rightarrow v = \sqrt{\frac{2 \times 2 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}}$$

$$= \sqrt{\frac{6.4}{9.1}} \times 10^6 \text{ m s}^{-1} = 0.84 \times 10^6 \text{ m s}^{-1}$$

$$= 8.4 \times 10^5 \text{ m s}^{-1}$$

4. (a): Here, $T_{1/2} = 4.47 \times 10^3 \text{ yr}$, $N = \frac{60}{100} N_0$
 $\therefore \frac{N}{N_0} = \left(\frac{1}{2}\right)^n \Rightarrow \frac{60}{100} = \left(\frac{1}{2}\right)^n$ or $2^n = \frac{10}{6}$
 $\Rightarrow n \log 2 = \log 10 - \log 6 = 1 - 0.778 = 0.222$
 $\therefore n = \frac{0.222}{\log 2} = \frac{0.222}{0.3} = 0.74$
Now, $t = nT_{1/2} = 0.74 \times 4.47 \times 10^3 \text{ yr} = 3.3 \times 10^3 \text{ yr}$.
5. (a): Here, $A = 5^\circ$, $\mu = 1.5$, $i = \theta$, $e = 0^\circ$
As the emergent ray is normal to the refracting surface of the prism
Hence, for a small angled prism,
 $\delta = (\mu - 1)A$,
 $\delta = (1.5 - 1)5^\circ = 2.5^\circ$
Since, $A + \delta = i + e$,
 $\Rightarrow 5^\circ + 2.5^\circ = \theta + 0^\circ$
or $\theta = 7.5^\circ$
6. (a): Diffraction effects will be observable only when width of the slit (a) is of the order of wavelength of light (λ).
Here, $a = 10^4 \text{ \AA}$ and $\lambda \sim 10^3 \text{ \AA}$
7. (c): Useful intensity for the emission of electron is
 $I' = 1\% \text{ of } I = \frac{1}{100} \times 39.6 = 0.396 \text{ W m}^{-2}$
Energy of each photon $= \frac{hc}{\lambda}$
 $= \frac{(6.64 \times 10^{-34}) \times (3 \times 10^8)}{6000 \times 10^{-10}} = 3.32 \times 10^{-19} \text{ J}$
No. of photoelectrons emitted per second per unit area
 $= \frac{0.396}{3.32 \times 10^{-19}} \approx 1.2 \times 10^{18}$
8. (d): As $r_n = \frac{n^2}{Z} a_0$, where n is the orbit number. For
 $^{257}_{100} \text{ Fm}$, $Z = 100$
 $\therefore r_5 = \frac{25}{100} a_0 = \frac{1}{4} a_0$
or $\frac{r_5}{r_0} = \frac{1}{4} \Rightarrow N = \frac{1}{4}$

9. (d)

10. (a): As, $y_0 = \frac{D}{d}(\mu - 1)t$

For same D , d and t , we have,

$$\frac{x}{\frac{3}{2}x} = \frac{(1.5 - 1)}{(\mu - 1)} \Rightarrow \frac{2}{3} = \frac{1}{2(\mu - 1)}$$

$$\Rightarrow \frac{1}{\mu - 1} = \frac{4}{3} \Rightarrow \mu - 1 = \frac{3}{4}$$

$$\therefore \mu = \frac{7}{4} = 1.75$$

11. (c): Given, ${}^2_1\text{H} + {}^2_1\text{H} \rightarrow {}^4_2\text{He} + Q$

The total binding energy of the deuterons

$$= 4 \times 1.15 = 4.60 \text{ MeV}$$

The total binding energy of alpha particle

$$= 4 \times 7.1 = 28.4 \text{ MeV}$$

\therefore The energy released in the process

$$= 28.4 - 4.60 = 23.8 \text{ MeV}$$

12. (c): For bright fringe, $n_1 \lambda_1 = n_2 \lambda_2$

$$\therefore n(7.5 \times 10^{-5}) = (n + 1)(5 \times 10^{-5})$$

$$\Rightarrow 2.5 \times 10^{-5}n = 5 \times 10^{-5}$$

$$\text{or } n = \frac{5.0 \times 10^{-5}}{2.5 \times 10^{-5}} = 2.$$

13. (a): As for a telescope magnifying power, $M = \frac{f_0}{f_e}$

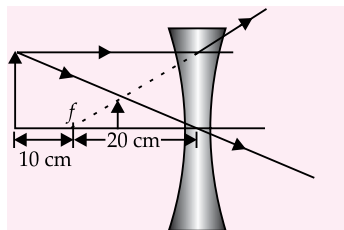
$$\therefore 9 = \frac{f_0}{f_e} \text{ or } f_0 = 9f_e$$

$$\text{Also, } L = f_0 + f_e \text{ or } 20 = f_0 + f_e$$

$$\text{or } 20 = 9f_e + f_e \text{ or } 20 = 10f_e \Rightarrow f_e = 2 \text{ cm}$$

$$\therefore f_0 = 9 \times 2 \text{ cm} = 18 \text{ cm}$$

14. (b): When an object is placed between $2f$ and f (focal length) of the diverging lens, the image is virtual, erect and diminished as shown in the ray diagram.



To calculate the distance of the image from the lens, we apply

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Rightarrow \frac{1}{-20} = \frac{1}{v} - \frac{1}{30}$$

$$\Rightarrow v = -\frac{(20)(30)}{20 + 30}$$

$$= -12 \text{ cm (to the left of the diverging lens.)}$$

15. (c): The angular momentum is given by

$$L = \sqrt{l(l+1)} \left(\frac{h}{2\pi} \right)$$

For $3d$ electron, $l = 2$.

$$\therefore L = \sqrt{2(3)} \left(\frac{h}{2\pi} \right) = \sqrt{6} \left(\frac{h}{2\pi} \right)$$

16. (d): Helium atom has 2 electrons. When one electron is removed, the remaining atom is hydrogen like atom, whose energy in first orbit is

$$E_1 = -(2)^2(13.6 \text{ eV}) = -54.4 \text{ eV}$$

Therefore, to remove the second electron from the atom, the additional energy of 54.4 eV is required.

Hence, total energy required to remove both the electrons = 24.6 + 54.4 = 79.0 eV.

17. (b): According to lens maker's formula

$$\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

where μ_1 is the refractive index of the medium in which lens is placed and μ_2 is the refractive index of the material of the lens.

\therefore For a convex lens in air

$$\frac{1}{f_a} = \left(\frac{\mu_g}{\mu_a} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \quad \dots(i)$$

For the same convex lens in liquid

$$\frac{1}{f_l} = \left(\frac{\mu_g}{\mu_l} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \quad \dots(ii)$$

Dividing (i) by (ii), we get

$$\frac{f_l}{f_a} = \frac{\mu_g - \mu_a}{\mu_a} \times \frac{\mu_l}{\mu_g - \mu_l}$$

$$\text{Here, } f_l = 0.150 + 0.225 = 0.375 \text{ m}$$

Substituting the given values, we get

$$\frac{0.375}{0.15} = \frac{\left(\frac{3}{2} - 1 \right)}{1} \times \frac{\mu_l}{\left(\frac{3}{2} - \mu_l \right)}$$

$$\Rightarrow \frac{5}{2} = \frac{\mu_l}{3 - 2\mu_l}$$

$$\text{or } 5(3 - 2\mu_l) = 2\mu_l$$

$$12\mu_l = 15$$

$$\mu_l = \frac{15}{12} = \frac{5}{4}$$

18. (d): Let I_0 be the intensity of unpolarized light, then intensity of light from first transparent thin plate of a polaroid is

$$I = \frac{I_0}{2}$$

Now this light will pass through the second similar plate whose axis is inclined at an angle of 30° to that of first plate.

According to Malus law, the intensity of emerging light is

$$I' = I \cos^2 30^\circ = \frac{I_0}{2} \left(\frac{\sqrt{3}}{2} \right)^2 = \frac{3}{8} I_0 \quad \therefore \frac{I'}{I_0} = \frac{3}{8}$$

19. (b): According to mirror formula,

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

Differentiating with respect to t , we get

$$\therefore -\frac{1}{u^2} \frac{du}{dt} - \frac{1}{v^2} \frac{dv}{dt} = 0 \quad (\because f \text{ is constant})$$

$$\text{or } \frac{dv}{dt} = -\left(\frac{v}{u}\right)^2 \frac{du}{dt}$$

$$v_i = -\left(\frac{v}{u}\right)^2 v_o \quad \left(\because \frac{dv}{dt} = v_i \text{ and } \frac{du}{dt} = v_o \right)$$

Substituting the given values, we get

$$v_i = -\left(\frac{10}{30}\right)^2 \times 9 = -1 \text{ m s}^{-1}$$

$$|v_i| = 1 \text{ m s}^{-1}$$

20. (c): According to Einstein's photoelectric equation

$$h\nu = h\nu_0 + \text{KE}_{\text{max}}$$

$$\text{For first experiment, } h \times 4 \times 10^{15} = h\nu_0 + x \quad \dots(i)$$

$$\text{For second experiment, } h \times 6 \times 10^{15} = h\nu_0 + 3x \dots(ii)$$

Subtracting equation (i) from (ii), we get,

$$2x = h \times 2 \times 10^{15} \text{ or } x = h \times 10^{15}$$

Putting in equation (i)

$$h\nu_0 = h \times 4 \times 10^{15} - h \times 10^{15} = 3 \times h \times 10^{15}$$

$$\text{or } \nu_0 = 3 \times 10^{15} \text{ Hz.}$$

21. (a)

22. (d): The effective path in air is increased by $(\mu - 1)t$ due to introduction of a transparent medium of refractive index μ and thickness t . Accordingly, the central fringe shifts to a new position.

The central fringe is bright or dark depends on the initial phase difference between the two coherent sources.

23. (a): Both the parts will be biconvex lenses, each of focal length 10 cm.

Focal length depends on the radii of curvatures of the two surfaces.

24. (c): Radius of Li^{++} ion in its ground state, i.e.,

$$r_0 = \frac{\epsilon_0 h^2}{\pi m Z e^2}$$

$$\text{As Bohr radius, } a_0 = \frac{\epsilon_0 h^2}{\pi m e^2} \Rightarrow r_0 = \frac{a_0}{Z} = \frac{53 \text{ pm}}{3} \approx 18 \text{ pm} \\ (\because \text{for Li, } Z = 3)$$

25. (b): During an elastic collision between two particles, the maximum kinetic energy is transferred from one particle to the other when they have the same mass. Consequently, a neutron loses all of its kinetic energy when it collides head-on with a proton, in analogy with the collision between a moving billiard ball and a stationary one. For this reason, materials which are abundant in hydrogen such as paraffin and water, are good moderators for neutrons.

26. (d): The path difference produced by a sheet

$$\Delta x = (\mu - 1)t$$

According to the given condition (for minimum thickness)

$$(\mu - 1)t = \frac{\lambda}{2}$$

$$\therefore t = \frac{\lambda}{2(\mu - 1)} = \frac{6600 \times 10^{-10}}{2(1.6 - 1)} = 5500 \text{ \AA.}$$

27. (c) 28. (c)

29. (a): Given, $A = 30^\circ$, $i_1 = 60^\circ$

As the ray retraces its path on reflection at the silvered face therefore,

$$i_2 = 0, r_2 = 0$$

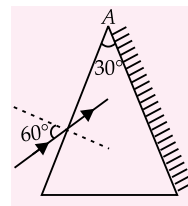
$$\text{As } r_1 + r_2 = A$$

$$\therefore r_1 + 0 = 30^\circ$$

$$\text{or } r_1 = 30^\circ$$

$$\mu = \frac{\sin i_1}{\sin r_1} = \frac{\sin 60^\circ}{\sin 30^\circ}$$

$$= \frac{\sqrt{3}/2}{1/2} = \sqrt{3}.$$



30. (b): For the relaxed eye, magnifying power is

$$M = \frac{v_o}{u_o} \frac{D}{f_e}$$

$$\therefore -45 = -\frac{v_o}{u_o} \times \frac{25}{5} \text{ or } \frac{v_o}{u_o} = 9.$$

For objective lens, image is real.

$$\therefore v_o = +v_o, u_o = -\frac{v_o}{9}.$$

Given, $f_o = 1 \text{ cm.}$

$$\text{From } \frac{1}{v_o} - \frac{1}{u_o} = \frac{1}{f_o}$$

$$\frac{1}{v_o} + \frac{9}{v_o} = \frac{1}{1}; v_o = 10 \text{ cm.}$$

$$\text{Length of the tube} = v_o + f_e = 10 + 5 = 15 \text{ cm}$$





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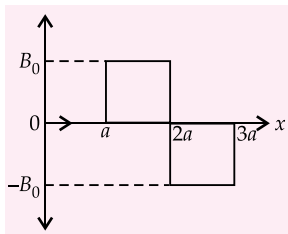
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Questions for Medical/ Engineering Entrance Exams

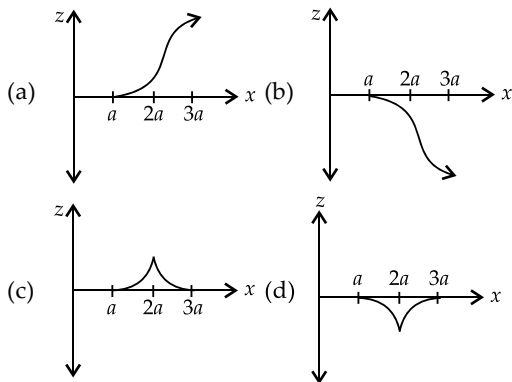
Moving Charges and Magnetism

- Which one of the following is not correct about Lorentz Force?
 - In presence of electric field $\vec{E}(r)$ and magnetic field $\vec{B}(r)$ the force on a moving electric charge is $\vec{F} = q[\vec{E}(r) + v \times \vec{B}(r)]$.
 - The force, due to magnetic field on a negative charge is opposite to that on a positive charge.
 - The force due to magnetic field become zero if velocity and magnetic field are parallel or anti-parallel.
 - For a static charge the magnetic force is maximum.
- A 4 A current carrying loop consists of three identical quarter circles of radius 5 cm lying in the positive quadrants of the x - y , y - z and z - x planes with their centres at the origin joined together, value of \vec{B} at the origin is
 - $\frac{\mu_0}{10}(\hat{i} + \hat{j} - \hat{k})$ T
 - $\frac{\mu_0}{10}(-\hat{i} + \hat{j} + \hat{k})$ T
 - $\frac{\mu_0}{5}(\hat{i} + \hat{j} + \hat{k})$ T
 - $10\mu_0(\hat{i} + \hat{j} + \hat{k})$ T
- A long straight wire carrying current of 30 A rests on a table. Another wire AB of length 1 m, mass 3 g carries the same current but in the opposite direction, the wire AB is free to slide up and down. The height upto which AB will rise is
 - 0.6 cm
 - 0.7 cm
 - 0.4 cm
 - 0.5 cm
- A circular coil of radius 10 cm having 100 turns carries a current of 3.2 A. The magnetic field at the center of the coil is
 - 2.01×10^{-3} T
 - 5.64×10^{-3} T
 - 2.64×10^{-4} T
 - 5.01×10^{-4} T
- An electron having momentum 2.4×10^{-23} kg m s⁻¹ enters a region of uniform magnetic field of 0.15 T. The field vector makes an angle of 30° with the initial velocity vector of the electron. The radius of the helical path of the electron in the field shall be
 - 2 mm
 - 1 mm
 - $\frac{\sqrt{3}}{2}$ mm
 - 0.5 mm
- An electron is moving in a cyclotron at a speed of 3.2×10^7 m s⁻¹ in a magnetic field of 5×10^{-4} T perpendicular to it. What is the frequency of this electron?
($e = 1.6 \times 10^{-19}$ C, $m_e = 9.1 \times 10^{-31}$ kg)
 - 1.4×10^5 Hz
 - 1.4×10^7 Hz
 - 1.4×10^6 Hz
 - 1.4×10^9 Hz
- A proton is accelerating on a cyclotron having oscillating frequency of 11 MHz in external magnetic field of 1 T. If the radius of its dees is 55 cm, then its kinetic energy (in MeV) is ($m_p = 1.67 \times 10^{-27}$ kg, $e = 1.6 \times 10^{-19}$ C)
 - 13.36
 - 12.52
 - 15.89
 - 14.49
- A straight wire having mass of 1.2 kg and length of 1 m carries a current of 5A. If the wire is suspended in mid-air by a uniform horizontal magnetic field, then the magnitude of field is
 - 0.65 T
 - 1.53 T
 - 2.4 T
 - 3.2 T

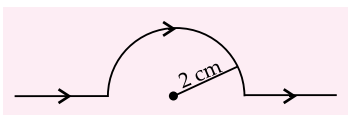
9. A magnetic field $\vec{B} = B_0 \hat{j}$ exists in the region $a < x < 2a$ and $\vec{B} = -B_0 \hat{j}$ in the region $2a < x < 3a$, where B_0 is a positive constant. A positive point charge moving with a velocity



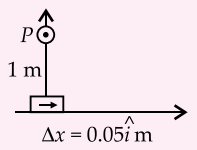
$\vec{v} = v_0 \hat{i}$ where v_0 is a positive constant, enters the magnetic field at $x = a$. The trajectory of the charge in this region can be like,



10. A straight wire carrying a current of 13 A is bent into a semi-circular arc of radius 2 cm as shown in figure. The magnetic field is 1.5×10^{-4} T at the centre of arc, then the magnetic field due to straight segment is

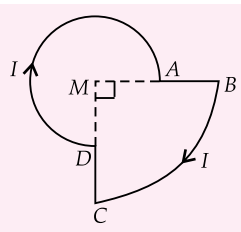


- (a) 1.5×10^{-4} T (b) 2.5×10^{-4} T
(c) zero (d) 3.0×10^{-4} T
11. An element of $0.05 \hat{i}$ m is placed at the origin as shown in figure which carries a large current of 10 A. The magnetic field at a distance of 1 m in perpendicular direction is
- (a) 4.5×10^{-8} T (b) 5.5×10^{-8} T
(c) 5.0×10^{-8} T (d) 7.5×10^{-8} T
12. Two particles of equal charges after being accelerated through the same potential difference enter in a uniform transverse magnetic field and describe circular paths of radii R_1 and R_2 . Then the ratio of their respective masses (M_1/M_2) is
- (a) R_1/R_2 (b) $(R_1/R_2)^2$
(c) R_2/R_1 (d) $(R_2/R_1)^2$



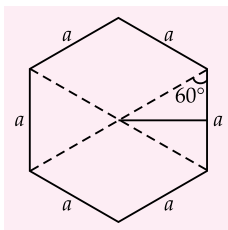
13. A solenoid of length 0.6 m has a radius of 2 cm and is made up of 600 turns. If it carries a current of 4 A, then the magnitude of the magnetic field inside the solenoid is
- (a) 6.024×10^{-3} T (b) 8.024×10^{-3} T
(c) 5.024×10^{-3} T (d) 7.024×10^{-3} T
14. The inner and outer radius of a toroid core are 28 cm and 29 cm respectively and around the core 3700 turns of a wire are wound. If the current in the wire is 10 A, then the magnetic field inside the core of the toroid is
- (a) 2.60×10^{-2} T (b) 2.60×10^{-3} T
(c) 4.52×10^{-2} T (d) 4.52×10^{-3} T
15. The horizontal component of earth's magnetic field at a certain place is 3.0×10^{-5} T and having a direction from the geographic south to geographic north. The force per unit length on a very long straight conductor carrying a steady current of 1.2 A in east to west direction is
- (a) 3.0×10^{-5} N m $^{-1}$ (b) 4.5×10^{-5} N m $^{-1}$
(c) 3.6×10^{-5} N m $^{-1}$ (d) 5.5×10^{-5} N m $^{-1}$
16. A circular coil of wire consisting of 100 turns each of radius 9 cm carries a current of 0.4 A. The magnitude of magnetic field at the centre of the coil is
- (a) 2.4×10^{-4} T (b) 3.2×10^{-4} T
(c) 2.79×10^{-4} T (d) 3.92×10^{-4} T
17. A circular coil of 70 turns and radius 5 cm carrying a current of 8 A is suspended vertically in a uniform horizontal magnetic field of magnitude 1.5 T. The field lines make an angle of 30° with the normal of the coil then the magnitude of the counter torque that must be applied to prevent the coil from turning is
- (a) 33 N m (b) 3.3 N m
(c) 3.3×10^{-2} N m (d) 3.3×10^{-4} N m
18. The electric current in a circular coil of two turns produced a magnetic induction of 0.2 T at its centre. The coil is unwound and then rewound into a circular coil of four turns. If same current flows in the coil, the magnetic induction at the centre of the coil now is
- (a) 0.2 T (b) 0.4 T
(c) 0.6 T (d) 0.8 T
19. A circular coil of 25 turns and radius 12 cm is placed in a uniform magnetic field of 0.5 T normal to the plane of the coil. If the current in the coil is 6 A then total torque acting on the coil is
- (a) zero (b) 3.4 N m
(c) 3.8 N m (d) 4.4 N m

20. A current I is flowing through a loop. The direction of the current and the shape of the loop are as shown in the figure. The magnetic field at the centre of the loop is $\frac{\mu_0 I}{R}$ times



(Here, $MA = R$, $MB = 2R$, $\angle DMA = 90^\circ$)

- (a) $\frac{5}{16}$, out of the plane of the paper.
 (b) $\frac{5}{16}$, into the plane of the paper.
 (c) $\frac{7}{16}$, out of the plane of the paper.
 (d) $\frac{7}{16}$, into the plane of the paper.
21. What is the correct value of Bohr magneton?
 (a) $8.99 \times 10^{-24} \text{ A m}^2$ (b) $9.27 \times 10^{-24} \text{ A m}^2$
 (c) $5.66 \times 10^{-24} \text{ A m}^2$ (d) $9.27 \times 10^{-28} \text{ A m}^2$
22. If an electron is projected with uniform velocity along the axis of a current carrying long solenoid, then
 (a) the electron will be accelerated along the axis.
 (b) the electron will continue to move with uniform velocity along the axis of the solenoid.
 (c) the electron path will be circular about the axis.
 (d) the electron will experience a force at 45° to the axis and hence executes a helical path.
23. A circular current loop of magnetic moment \vec{M} is in an arbitrary orientation in an external magnetic field \vec{B} . The work done to rotate the loop through 45° about an axis perpendicular to its plane is
 (a) zero (b) $\frac{3MB}{\sqrt{2}}$
 (c) $\frac{MB}{\sqrt{2}}$ (d) $\sqrt{2}MB$
24. A uniform conducting wire of length $18a$ and resistance R is wound up as current carrying coil in the shape of a regular hexagon of sides a . If the coil is connected to a voltage source V_0 , then the magnetic moment of coil is



- (a) $6\sqrt{3} \frac{V_0}{R} a^2 \text{ A m}^2$ (b) $9 \frac{\sqrt{3}}{2} \frac{V_0}{R} a^2 \text{ A m}^2$
 (c) $\frac{7\sqrt{3}}{2} \frac{V_0}{R} a^2 \text{ A m}^2$ (d) $\frac{11\sqrt{3}}{2} \frac{V_0}{R} a^2 \text{ A m}^2$

25. If electron moving with velocity \vec{v} produces a magnetic field \vec{B} , then
 (a) the direction of field \vec{B} will be same in the direction of velocity \vec{v} .
 (b) the direction of field \vec{B} will be opposite to the direction of velocity \vec{v} .
 (c) the direction of field \vec{B} will be perpendicular to the direction of velocity \vec{v} .
 (d) the direction of field \vec{B} does not depend upon the direction of velocity \vec{v} .

26. A long straight wire in the horizontal plane carries a current of 75 A in north to south direction, magnitude and direction of field B at a point 3 m east of the wire is
 (a) $4 \times 10^{-6} \text{ T}$, vertical up
 (b) $5 \times 10^{-6} \text{ T}$, vertical down
 (c) $5 \times 10^{-6} \text{ T}$, vertical up
 (d) $4 \times 10^{-6} \text{ T}$, vertical down

27. A proton and an α -particle enter in a uniform magnetic field perpendicularly with same speed.

The ratio of time periods of both particle $\left(\frac{T_p}{T_\alpha} \right)$ will be

- (a) 1 : 2 (b) 1 : 3
 (c) 2 : 1 (d) 3 : 1
28. A conductor of length 2 m carrying current 2 A is held parallel to an infinitely long conductor carrying current of 12 A at a distance of 100 mm, the force on small conductor is
 (a) $8.6 \times 10^{-5} \text{ N}$ (b) $6.6 \times 10^{-5} \text{ N}$
 (c) $7.6 \times 10^{-5} \text{ N}$ (d) $9.6 \times 10^{-5} \text{ N}$
29. A short bar magnet has a magnetic moment of 0.65 J T^{-1} , then the magnitude and direction of the magnetic field produced by the magnet at a distance 8 cm from the centre of magnet on the axis is
 (a) $2.5 \times 10^{-4} \text{ T}$, along NS direction
 (b) $2.5 \times 10^{-4} \text{ T}$, along SN direction
 (c) $4.5 \times 10^{-4} \text{ T}$, along NS direction
 (d) $4.5 \times 10^{-4} \text{ T}$, along SN direction

30. A galvanometer of resistance 50Ω is connected to a battery of 3 V along with a resistance of 2950Ω in series. A full scale deflection of 30 divisions is obtained in the galvanometer. In order to reduce this deflection to 20 divisions, the resistance in series should be
 (a) 6050 Ω (b) 4450 Ω
 (c) 5050 Ω (d) 5550 Ω

SOLUTION

1. (d): If charge is not moving then the magnetic force is zero.

$$\text{Since } \vec{F}_m = q(\vec{v} \times \vec{B})$$

As $\vec{v} = 0$, for stationary charge

$$\therefore \vec{F}_m = 0$$

2. (d): As $\vec{B} = \vec{B}_{xy} + \vec{B}_{yz} + \vec{B}_{zx}$... (i)

$$\text{where } \vec{B}_{xy} = \frac{\mu_0}{4\pi R} I \hat{k}, \vec{B}_{yz} = \frac{\mu_0}{4\pi R} I \hat{i}, \vec{B}_{zx} = \frac{\mu_0}{4\pi R} I \hat{j},$$

Substituting these values in equation (i) we get,

$$\vec{B} = \frac{\mu_0}{4\pi R} I [\hat{i} + \hat{j} + \hat{k}]$$

Here, $\theta = \frac{\pi}{2}$, $I = 4$ A and $R = 5$ cm = 5×10^{-2} m

$$\therefore \vec{B} = \frac{\mu_0}{4\pi} \times \frac{4}{5 \times 10^{-2}} \times \frac{\pi}{2} (\hat{i} + \hat{j} + \hat{k})$$

$$= 10\mu_0 (\hat{i} + \hat{j} + \hat{k}) \text{ T}$$

3. (a): Here $I_1 = I_2 = 30$ A, $l = 1$ m,

$$m = 3 \text{ g} = 3 \times 10^{-3} \text{ kg}$$

In equilibrium position,

$$mg = \frac{\mu_0}{4\pi} \frac{2I_1 I_2 l}{h}$$

$$h = \frac{\mu_0}{4\pi} \times \frac{2I_1 I_2 l}{mg} = 10^{-7} \times \frac{2 \times 30 \times 30 \times 1}{3 \times 10^{-3} \times 10}$$

$$= 6 \times 10^{-3} \text{ m} = 0.6 \text{ cm}$$

4. (a): As $B = \frac{\mu_0 NI}{2R}$, Here $N = 100$, $I = 3.2$ A,

and $R = 10$ cm = 10×10^{-2} m = 0.1 m

$$\therefore B = \frac{4\pi \times 10^{-7} \times 100 \times 3.2}{2 \times 0.1} = 2.01 \times 10^{-3} \text{ T}$$

5. (d): The radius of the helical path of the electron in the uniform magnetic field is

$$r = \frac{mv_{\perp}}{eB} = \frac{mv \sin \theta}{eB} = \frac{(2.4 \times 10^{-23} \text{ kg m s}^{-1}) \times \sin 30^\circ}{(1.6 \times 10^{-19} \text{ C}) \times (0.15 \text{ T})}$$

$$= 5 \times 10^{-4} \text{ m} = 0.5 \times 10^{-3} \text{ m} = 0.5 \text{ mm}$$

6. (b): Here, $v = 3.2 \times 10^7$ m s⁻¹, $B = 5 \times 10^{-4}$ T
The frequency of electron is

$$\nu = \frac{eB}{2\pi m_e} = \frac{1.6 \times 10^{-19} \times 5 \times 10^{-4}}{2 \times 3.14 \times 9.1 \times 10^{-31}}$$

$$= 1.4 \times 10^7 \text{ Hz} = 14 \text{ MHz}$$

7. (d): Here, $\nu_c = 11$ MHz = 11×10^6 Hz

$$B = 1 \text{ T}, R = 55 \text{ cm} = 55 \times 10^{-2} \text{ m},$$

$$e = 1.6 \times 10^{-19} \text{ C and } m_p = 1.67 \times 10^{-27} \text{ kg},$$

$$\therefore \text{K.E.} = \frac{q^2 B^2 R^2}{2m} = \frac{(1.6 \times 10^{-19})^2 \times (1)^2 \times (55 \times 10^{-2})^2}{2 \times 1.67 \times 10^{-27}}$$

$$= 23.19 \times 10^{-13} \text{ J}$$

$$= \frac{23.19 \times 10^{-13}}{1.6 \times 10^{-19}} \text{ eV} = 14.49 \times 10^6 \text{ eV} = 14.49 \text{ MeV}$$

8. (c): For mid-air suspension the upward force F on wire due to magnetic field B must be balanced by the force due to gravity,

$$\Rightarrow \begin{aligned} l B &= mg \\ B &= \frac{mg}{l} \end{aligned}$$

Here, $m = 1.2$ kg, $g = 10$ m s⁻², $I = 5$ A, $l = 1$ m

$$\therefore B = \frac{1.2 \times 10}{5 \times 1} = 2.4 \text{ T}$$

9. (a): Force on the charged particle in a magnetic field, $\vec{F} = q(\vec{v} \times \vec{B})$.

In the region $a < x < 2a$, the force on the charged particle, must be in the direction of $\hat{i} \times \hat{j}$, that is, in $+z$ direction, which is vertically upwards. And in the region $2a < x < 3a$, the force on the charged particle will be in $-z$ direction, which is vertically downwards.

For $a < x < 2a$, path will be concave upward.

For $2a < x < 3a$, path will be concave downward. Moreover, there should not be any kink in the path at $x = 2a$.

Hence graph (a) is correct.

10. (c): Since dl and r for each element of the straight segments are either parallel or antiparallel. Therefore

$$\vec{dl} \times \vec{r} = 0$$

Hence, B due to straight segment is also zero.

11. (c): $dB = \frac{\mu_0}{4\pi} \frac{Idl \sin \theta}{r^2}$

Here, $dl = \Delta x = 0.05$ m, $I = 10$ A, $r = 1$ m

and $\sin \theta = \sin 90^\circ = 1$,

$$\therefore dB = 10^{-7} \times \frac{10 \times 0.05 \times 1}{(1)^2}$$

$$= 0.50 \times 10^{-7} = 5.0 \times 10^{-8} \text{ T}$$

12. (b): $qV = \frac{1}{2} Mv^2$ or $v = \sqrt{\frac{2qV}{M}}$... (i)

$$\text{And } Bqv = \frac{Mv^2}{R}$$

$$\text{or } Bq = \frac{Mv}{R} = \frac{M}{R} \sqrt{\frac{2qV}{M}} = \frac{\sqrt{2qVM}}{R} \quad (\text{Using (i)})$$

$$\text{or } M = \frac{B^2 q^2 R^2}{2qV} = \frac{B^2 q R^2}{2V}$$

$$\therefore M \propto R^2 \quad \left(\because B, q \text{ and } V \text{ are same for the given two particles} \right)$$

$$\text{Hence } \frac{M_1}{M_2} = \left(\frac{R_1}{R_2} \right)^2$$

13. (c) : Here, $l = 0.6$ m and $r = 0.02$ m

$$n = \frac{600}{0.6} = 1000 \text{ turns / m, } I = 4 \text{ A}$$

$$\therefore \frac{l}{r} = 30 \text{ i.e. } l \gg r$$

Hence, we can use long solenoid formula,

$$\therefore B = \mu_0 n I = 4\pi \times 10^{-7} \times 10^3 \times 4 = 50.24 \times 10^{-4} = 5.024 \times 10^{-3} \text{ T}$$

14. (a) : The number of turns per unit length for the given toroid, $n = \frac{N}{2\pi r_{av}}$

The average radius of toroid,

$$r_{av} = \frac{28 + 29}{2} = 28.5 \text{ cm} = 28.5 \times 10^{-2} \text{ m}$$

$$\therefore n = \frac{3700}{2 \times 3.14 \times 28.5 \times 10^{-2}} = 2067.27 \approx 2067 \text{ turns/m.}$$

$$\text{Now, } B = \mu_0 n I = 4\pi \times 10^{-7} \times 2067 \times 10 = 259615.2 \times 10^{-7} \text{ T} = 2.60 \times 10^{-2} \text{ T}$$

15. (c) : Since $\vec{F} = I\vec{l} \times \vec{B}$

$$\therefore F = IlB \sin \theta$$

$$\text{Now force per unit length, } f = \frac{F}{l} = IB \sin \theta$$

When the current is flowing from east to west then $\theta = 90^\circ$, hence

$$f = IB \sin 90^\circ = 1.2 \times 3 \times 10^{-5} \times 1 = 3.6 \times 10^{-5} \text{ N m}^{-1}$$

16. (c) : Here, $N = 100$

$$R = 9 \text{ cm} = 9 \times 10^{-2} \text{ m and } I = 0.4 \text{ A}$$

$$\begin{aligned} \text{Now, } B &= \frac{\mu_0 N I}{2R} = \frac{2\pi \times 10^{-7} \times 100 \times 0.4}{9 \times 10^{-2}} \\ &= \frac{2 \times 3.14 \times 0.4}{9} \times 10^{-3} \\ &= 0.279 \times 10^{-3} \text{ T} = 2.79 \times 10^{-4} \text{ T} \end{aligned}$$

17. (b) : Here, $N = 70$

$$r = 5 \text{ cm} = 5 \times 10^{-2} \text{ m, } I = 8 \text{ A}$$

$$B = 1.5 \text{ T and } \theta = 30^\circ$$

The counter torque to prevent the coil from turning will be equal and opposite to the torque acting on the coil,

$$\begin{aligned} \therefore \tau &= NIAB \sin \theta = NI\pi r^2 B \sin 30^\circ \\ &= 70 \times 8 \times 3.14 \times (5 \times 10^{-2})^2 \times 1.5 \times \frac{1}{2} \\ &= 3.3 \text{ N m} \end{aligned}$$

18. (d) : When there are two turns in the coil, then

$$l = 2 \times 2\pi r_1 \text{ or } r_1 = \frac{l}{4\pi}$$

$$\text{then } B_1 = \frac{\mu_0 N_1 I}{2r_1} = \frac{\mu_0 \times 2 \times I}{2 \times (l/4\pi)} = \frac{\mu_0 4\pi I}{l}$$

When there are four turns in the coil, then

$$l = 4 \times 2\pi r_2 \text{ or } r_2 = \frac{l}{8\pi}$$

$$\text{Then } B_2 = \frac{\mu_0 N_2 I}{2r_2} = \frac{\mu_0 \times 4 \times I}{2 \times (l/8\pi)} = \frac{\mu_0 16\pi I}{l}$$

$$\Rightarrow \frac{B_1}{B_2} = \frac{4}{16} = \frac{1}{4} \text{ or } B_2 = 4B_1 = 4 \times 0.2 \text{ T} = 0.8 \text{ T}$$

19. (a) : The torque acting on the coil

$$|\vec{\tau}| = |\vec{m} \times \vec{B}| = mB \sin \theta$$

Here the circular coil is placed normal to the direction of magnetic field then the angle between the direction of magnetic moment (\vec{m}) and magnetic field (\vec{B}) is zero, then

$$\tau = mB \sin \theta = mB \sin 0 = 0 \Rightarrow \therefore \tau = 0$$

20. (d) : Magnetic field at the centre M due to current through the curved portion DA is

$$\begin{aligned} B_1 &= \frac{\mu_0 I}{4\pi R} \times \left(\frac{3\pi}{2} \right) \text{ into the plane of the paper} \\ &= \frac{3\mu_0 I}{8R} \text{ into the plane of the paper} \end{aligned}$$

Magnetic field at the centre M due to current through the straight portion AB is $B_2 = 0$ (\because point M lies on the axis of the straight portion AB)

Magnetic field at the centre M due to current through the curved portion BC is

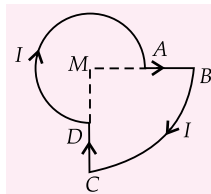
$$\begin{aligned} B_3 &= \frac{\mu_0 I}{4\pi(2R)} \times \frac{\pi}{2} \text{ into the plane of the paper} \\ &= \frac{\mu_0 I}{16R} \text{ into the plane of the paper} \end{aligned}$$

Magnetic field at the centre M due to current through the straight portion CD is $B_4 = 0$

(\because point M lies on the axis of the straight portion CD)

The resultant magnetic field at M is

$$\begin{aligned} B &= B_1 + B_2 + B_3 + B_4 \\ &= \frac{3\mu_0 I}{8R} + 0 + \frac{\mu_0 I}{16R} + 0 = \frac{3\mu_0 I}{8R} + \frac{\mu_0 I}{16R} \\ &= \frac{7\mu_0 I}{16R} \text{ into the plane of the paper} \end{aligned}$$



21. (b): Bohr Magnetron, $\mu_B = (\mu_l)_{\min} = \frac{e}{4\pi m_e} h$

Here, $e = 1.6 \times 10^{-19}$ C

$$h = 6.64 \times 10^{-34} \text{ J s}$$

$$m_e = 9.1 \times 10^{-31} \text{ kg}$$

$$\therefore \mu_B = \frac{1.6 \times 10^{-19} \times 6.64 \times 10^{-34}}{4 \times 3.14 \times 9.1 \times 10^{-31}} \\ = 9.27 \times 10^{-24} \text{ A m}^2$$

22. (b): Let the electron (e) is projected with a uniform velocity (v) in a uniform magnetic field B . The magnitude of force on it is

$$|\vec{F}| = -e |\vec{v} \times \vec{B}| = -evB \sin \theta$$

$$\text{As } \theta = 0, |\vec{F}| = -evB \sin \theta = 0$$

Hence the electron will continue to move with a uniform velocity along the axis of the solenoid.

23. (a): Due to rotation of the loop through 45° about the axis perpendicular to the plane of loop there will be no change in the angle between magnetic moment \vec{M} and magnetic field, therefore $\theta_1 = 45^\circ$ and $\theta_2 = 45^\circ$ and work done $W = MB(\cos \theta_1 - \cos \theta_2)$
 $= MB(\cos 45^\circ - \cos 45^\circ) = 0$

24. (b): In the regular hexagon, if each arm length is a , then the number of turns in the given shape,

$$N = \frac{18a}{6a} = 3$$

The area of given shape,

$$A = \frac{3\sqrt{3}a^2}{2}$$

Now magnetic moment $M = NIA$

$$= 3 \times I \times \frac{3\sqrt{3}}{2} a^2 = \frac{9\sqrt{3}}{2} \frac{V_o a^2}{R} \text{ A m}^2 \quad (\because I = \frac{V_o}{R})$$

25. (c): According to Biot Savart's law, the magnetic field

$$\vec{B} = \frac{\mu_o}{4\pi} \cdot \frac{q(\vec{v} \times \vec{r})}{r^3}$$

The direction of \vec{B} will be along $\vec{v} \times \vec{r}$, i.e. perpendicular to the plane containing \vec{v} and \vec{r} .

26. (c): From ampere circuital law,

$$\oint \vec{B} \cdot d\vec{l} = \mu_o I_{\text{enc}}$$

$$B \times 2\pi R = \mu_o I_{\text{enc}}$$

$$B = \frac{\mu_o I_{\text{enc}}}{2\pi R} = 2 \times 10^{-7} \times \frac{75}{3} = 5 \times 10^{-6} \text{ T}$$

The direction of field at the given point will be vertical up determined by the screw rule or right hand rule.

27. (a): The time period of revolution of a charged particle in a magnetic field is

$$T = \frac{2\pi m}{Bq}$$

For proton, $m_p = m$, $q_p = q$

$$\therefore T_p = \frac{2\pi m}{Bq}$$

Now, for α -particle,

$$m_\alpha = 4m, q_\alpha = 2q$$

$$\therefore T_\alpha = \frac{2\pi(4m)}{B(2q)} = 2 \left(\frac{2\pi m}{Bq} \right) = 2T_p \text{ or } \frac{T_p}{T_\alpha} = \frac{1}{2}$$

28. (d): Here, $I_1 = 2$ A, $I_2 = 12$ A

$$r = 100 \text{ mm} = 0.1 \text{ m}, l = 2 \text{ m}$$

The force per unit length on small conductor due to long conductor

$$f = \frac{\mu_o}{4\pi} \frac{2I_1 I_2}{r}$$

Now total force on length l of small conductor

$$F = f \times l = \frac{\mu_o}{4\pi} \frac{2I_1 I_2}{r} \times l \\ = \frac{10^{-7} \times 2 \times 2 \times 12 \times 2}{0.1} = 9.6 \times 10^{-5} \text{ N}$$

29. (b): Here, $M = 0.65 \text{ J T}^{-1}$ and $d = 8 \text{ cm} = 0.08 \text{ m}$

The field produced by magnet at axial point is given by

$$B = \frac{\mu_o}{4\pi} \frac{2M}{d^3} = 10^{-7} \times \frac{2 \times 0.65}{(0.08)^3} = 2.5 \times 10^{-4} \text{ T along SN.}$$

30. (b): Total initial resistance = $G + R$
 $= 50 \Omega + 2950 \Omega = 3000 \Omega$

$$\text{Current, } I = \frac{3 \text{ V}}{3000 \Omega} = 1 \times 10^{-3} \text{ A} = 1 \text{ mA}$$

If the deflection has to be reduced to 20 divisions, then current

$$I' = \frac{1 \text{ mA}}{30} \times 20 = \frac{2}{3} \text{ mA}$$

Let x be the effective resistance of the circuit,

$$3 \text{ V} = 3000 \Omega \times 1 \text{ mA} = x \Omega \times \frac{2}{3} \text{ mA}$$

$$\text{or } x = 3000 \times 1 \times \frac{3}{2} = 4500 \Omega$$

$$\therefore \text{Resistance to be added} = (4500 \Omega - 50 \Omega) \\ = 4450 \Omega$$

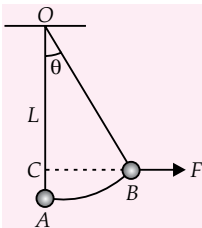
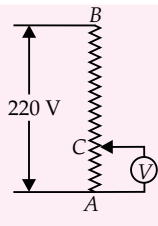




VERI- SIMILAR PRACTICE PAPER 2014

Exam on
4th May

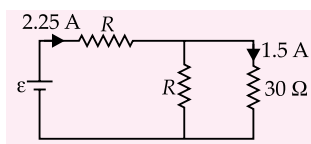
AIPMT Special

- Two cars are moving in the same direction with a speed of 30 km h^{-1} . They are separated from each other by 5 km. Third car moving in the opposite direction meets the two cars after an interval of 4 min. What is the speed of the third car?
(a) 30 km h^{-1} (b) 35 km h^{-1}
(c) 40 km h^{-1} (d) 45 km h^{-1}
- A 24 kg block resting on a floor has a rope tied to its top. The maximum tension, the rope can withstand without breaking is 310 N. The minimum time in which the block can be lifted a vertical distance of 4.6 m by pulling on the rope is
(a) 1.2 s (b) 2.9 s
(c) 1.7 s (d) 2.3 s
- An object of mass m is tied to a string of length L and a variable horizontal force is applied on it which starts at zero and gradually increases until the string makes an angle θ with the vertical. Work done by the force F is

(a) $mgL(1 - \sin\theta)$ (b) mgL
(c) $mgL(1 - \cos\theta)$ (d) $mgL(1 + \cos\theta)$
- A ball falls freely from a height of 45 m. When the ball is at a height of 25 m, it explodes into two equal pieces. One of them moves horizontally with a speed of 10 m s^{-1} . The distance between the two pieces when both strike the ground is
(a) 10 m (b) 20 m (c) 15 m (d) 30 m
- The ratio of energy required to raise a satellite to a height h above the earth's surface to that required to put it into the orbit is
(a) $h : R$ (b) $h : 2R$ (c) $2h : R$ (d) $R : h$
- A steel wire of length 20 cm and uniform cross-section 1 mm^2 is tied rigidly at both the ends. The temperature of the wire is altered from 40°C to 20°C . Coefficient of linear expansion of steel is $\alpha = 1.1 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$ and Y for steel is $2.0 \times 10^{11} \text{ N m}^{-2}$, the tension in the wire is
(a) $2.2 \times 10^6 \text{ N}$ (b) 16 N
(c) $8 \times 10^6 \text{ N}$ (d) 44 N
- A cylindrical vessel is filled with equal amounts by weights of mercury and water. The total height of the two layers is 29.2 cm. The pressure of the liquid at the bottom of the vessel is (Given, specific gravity of mercury is 13.6)
(a) 8 cm of Hg (b) 6 cm of Hg
(c) 4 cm of Hg (d) 2 cm of Hg.
- At what temperature, the average kinetic energy of translatory motion of a gas molecule will be equal to that of an electron accelerated through a potential difference of 10 V? (Take Boltzmann constant $k = 1.38 \times 10^{-23} \text{ J K}^{-1}$)
(a) $2.42 \times 10^3 \text{ K}$ (b) $7.73 \times 10^3 \text{ K}$
(c) $2.42 \times 10^4 \text{ K}$ (d) $7.73 \times 10^4 \text{ K}$
- An electric charge $10^{-3} \mu\text{C}$ is placed at the origin (0, 0) of X - Y co-ordinate system. Two points A and B are situated at $(\sqrt{2}, \sqrt{2})$ and (2, 0) respectively. The potential difference between the points A and B will be
(a) 4.5 V (b) 9 V (c) zero (d) 2 V
- A potential difference of 220 V is maintained across a 12000Ω rheostat AB as shown in figure. The voltmeter V has a resistance of 6000Ω and point C is at one fourth of the distance from A to B. What is the reading in the voltmeter ?

(a) 32 V (b) 36 V
(c) 40 V (d) 42 V

11. A straight horizontal conducting rod of length 50 cm and mass 50 g is suspended by two vertical wires at its ends. A current of 5.0 A is set up in the rod through the wires. What magnetic field should be set up normal to the conductor in order that the tension in the wires is zero?
(Take $g = 10 \text{ m s}^{-2}$)
(a) 0.05 T (b) 0.1 T
(c) 0.2 T (d) 0.5 T
12. A man of mass m squatting on the ground gets straight up and stands. The force of reaction of ground on the man during the process is
(a) constant and equal to mg in magnitude
(b) constant and greater than mg in magnitude
(c) variable but always greater than mg
(d) at first greater than mg , and later becomes equal to mg .
13. A uniform rod of length l and mass m is swinging freely about a horizontal axis passing through its end. Its maximum angular speed is ω . Its centre of mass rises to a maximum height of
(a) $\frac{1}{3} \frac{l^2 \omega^2}{g}$ (b) $\frac{1}{6} \frac{l \omega}{g}$
(c) $\frac{1}{2} \frac{l^2 \omega^2}{g}$ (d) $\frac{1}{6} \frac{l^2 \omega^2}{g}$
14. A magnet is suspended lengthwise from a spring and while it oscillates, the magnet moves in and out of the coil C connected to a galvanometer G. Then as the magnet oscillates,
(a) G shows no deflection
(b) G shows deflection on one side
(c) deflection of G to the left and right has constant amplitude
(d) deflection of G to the left and right has decreasing amplitude.
15. A $16 \mu\text{F}$ capacitor is charged to a 20 V potential. The battery is then disconnected and a pure 40 mH coil is connected across the capacitor so that LC oscillations are set up. The maximum current in the coil is
(a) 0.2 A (b) 40 mA
(c) 2 A (d) 0.4 A
16. In Young's double slit expt., the distance between two sources is 0.1 mm. The distance of the screen from the source is 20 cm. Wavelength of light used is 5460 \AA . The angular position of the first dark fringe is
(a) 0.08° (b) 0.16° (c) 0.20° (d) 0.32°
17. A concave lens of focal length 20 cm placed in contact with a plane mirror acts as a

- (a) convex mirror of focal length 10 cm
(b) concave mirror of focal length 40 cm
(c) concave mirror of focal length 60 cm
(d) concave mirror of focal length 10 cm

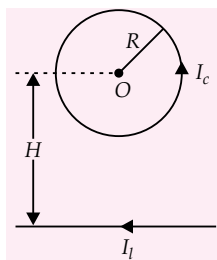
18. The weight based ratio of U^{238} and Pb^{226} in a sample of rock is 4 : 3. If the half life of U^{238} is 4.5×10^9 years, then the age of rock is
(a) 9.0×10^9 years (b) 6.3×10^9 years
(c) 4.5×10^9 years (d) 3.78×10^9 years
19. A bottle weighing 220 g and of area of cross-section 50 cm^2 and height 4 cm oscillates on the surface of water in vertical position. Its frequency of oscillation is
(a) 1.5 Hz (b) 2.5 Hz (c) 3.5 Hz (d) 4.5 Hz
20. A string of density 7.5 g cm^{-3} and area of cross-section 0.2 mm^2 is stretched under a tension of 20 N. When it is plucked at the midpoint, the speed of the transverse wave on the wire is
(a) 116 m s^{-1} (b) 40 m s^{-1}
(c) 200 m s^{-1} (d) 80 m s^{-1}
21. A parallel plate capacitor with air between the plates has a capacitance of 9 pF. The separation between the plates is d . The space between the plates is now filled with two dielectrics. One of the dielectrics has dielectric constant $K_1 = 3$ and thickness $d/3$ while the other one has dielectric constant $K_2 = 6$ and thickness $2d/3$. Capacitance of the capacitor is now
(a) 20.25 pF (b) 1.8 pF
(c) 45 pF (d) 40.5 pF
22. In the circuit shown, the value of



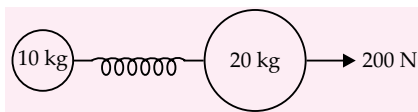
- (a) $R = 15 \Omega$ (b) $R = 30 \Omega$
(c) $\varepsilon = 36 \text{ V}$ (d) $\varepsilon = 180 \text{ V}$
23. Two resistances are connected in two gaps of a metre bridge. The balance point is 20 cm from the zero end. A resistance of 15Ω is connected in series with the smaller of the two. The null point shifts to 40 cm. The value of the smaller resistance in ohms is
(a) 3 (b) 6
(c) 9 (d) 12
24. A circular loop of a wire and a long straight wire carry currents I_c and I_l respectively as shown in figure. Assuming that these are placed in the

same plane, the magnetic field will be zero at the centre O of the loop, when separation H is

- (a) $\frac{I_l R}{I_c \pi}$
 (b) $\frac{I_c R}{I_l \pi}$
 (c) $\frac{I_c \pi}{I_l R}$
 (d) $\frac{I_l \pi}{I_c R}$



25. Two masses of 10 kg and 20 kg respectively are tied together by a massless spring. A force of 200 N is applied on a 20 kg mass as shown in figure. At the instant shown, the acceleration of 10 kg mass is 12 m s^{-2} , the acceleration of 20 kg mass is



- (a) 0
 (b) 10 m s^{-2}
 (c) 4 m s^{-2}
 (d) 12 m s^{-2}

26. The mass of the earth is increasing at the rate of 1 part in 5×10^{19} per day by the attraction of meteors falling normally on the earth's surface. Assuming that the density of earth is uniform, the rate of change of the period of rotation of the earth is

- (a) 2.0×10^{-20}
 (b) 2.66×10^{-19}
 (c) 4.33×10^{-18}
 (d) 5.66×10^{-17}

27. The twisting couple per unit twist for a solid cylinder of radius 3 cm is 0.1 N m. The twisting couple per unit twist, for a hollow cylinder of same material with outer and inner radius 5 cm and 4 cm respectively will be

- (a) 0.1 N m
 (b) 0.455 N m
 (c) 0.91 N m
 (d) 1.82 N m

28. A body takes 10 minutes to cool from 60°C to 50°C . If the temperature of surroundings is 25°C , then temperature of body after next 10 minutes will be

- (a) 48°C
 (b) 46°C
 (c) 40°C
 (d) 42.85°C

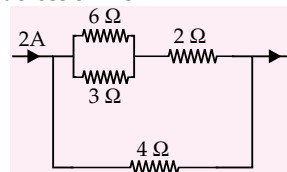
29. When a gas filled in a closed vessel is heated through 1°C , its pressure increases by 0.4%. The initial temperature of the gas was

- (a) 250 K
 (b) 2500 K
 (c) 250°C
 (d) 25°C

30. Two spherical conductors A and B of radii 1 mm and 2 mm are separated by a distance of 5 cm and are uniformly charged. If the spheres are connected by a conducting wire, then in equilibrium condition, the ratio of the magnitude of the electric fields at the surfaces of spheres A and B is

- (a) 4 : 1
 (b) 1 : 2
 (c) 2 : 1
 (d) 1 : 4

31. In the circuit shown in figure, the potential difference across 3Ω is



- (a) 2 V
 (b) 4 V
 (c) 8 V
 (d) 16 V

32. A thin disc having radius r and charge q distributed uniformly over the disc is rotated n rotations per second about its axis. The magnetic field at the centre of the disc is

- (a) $\frac{\mu_0 q n}{2r}$
 (b) $\frac{\mu_0 q n}{r}$
 (c) $\frac{3\mu_0 q n}{r}$
 (d) $\frac{3\mu_0 q n}{4r}$

33. A square loop of side a placed in the same plane as a long straight wire carrying a current I . The centre of the loop is at a distance r from the wire, where $r \gg a$,

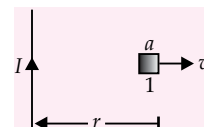


figure. The loop is moved away from the wire with a constant velocity v . The induced emf in the loop is

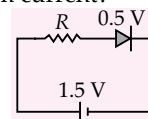
- (a) $\frac{\mu_0 I a v}{2\pi r}$
 (b) $\frac{\mu_0 I a^3 v}{2\pi r^3}$
 (c) $\frac{\mu_0 I v}{2\pi}$
 (d) $\frac{\mu_0 I a^2 v}{2\pi r^2}$

34. The focal length of a biconvex lens of refractive index 1.5 is 0.06 m. Radius of curvatures are in the ratio 1 : 2. Then radii of curvatures of two lens surface are

- (a) 0.045 m, 0.09 m
 (b) 0.09 m, 0.18 m
 (c) 0.04 m, 0.08 m
 (d) 0.06 m, 0.12 m

35. The diode used in the circuit shown in figure has a constant voltage drop at 0.5 V at all currents and a maximum power rating of 100 mW. What should be the value of the resistor R , connected in series with diode, for obtaining maximum current?

- (a) 6.76 Ω
 (b) 20 Ω
 (c) 5 Ω
 (d) 5.6 Ω



36. The wavelength of the first line of Lyman series for hydrogen atom is equal to that of the second line of Balmer series for a hydrogen like ion. The atomic number Z of hydrogen like ion is
(a) 2 (b) 3 (c) 4 (d) 1
37. Light from a hydrogen discharge tube is incident on the cathode of a photoelectric cell. The work function of the cathode surface is 4.2 eV. In order to reduce the photocurrent to zero, the voltage of the anode relative to the cathode must be made
(a) -4.2 V (b) -9.4 V
(c) -17.8 V (d) +9.4 V
38. The size of the image of an object, which is at infinity, as formed by a convex lens of focal length 30 cm is 2 cm. If a concave lens of focal length 20 cm is placed between the convex lens and the image at a distance of 26 cm from the convex lens, then the new size of the image is
(a) 1.25 cm (b) 2.5 cm
(c) 1.05 cm (d) 2 cm
39. Two point white dots are 1 mm apart on a black paper. They are viewed by an eye of pupil diameter 3 mm. Approximately, what is the maximum distance at which these dots can be resolved by the eye? (Take wavelength of light = 500 nm)
(a) 5 m (b) 1 m (c) 6 m (d) 3 m
40. The Earth is assumed to be a sphere of radius R . A platform is arranged at a height R from the surface of the Earth. The escape velocity of a body from the platform is fv , where v is the escape velocity from the surface of the Earth. The value of f is
(a) $1/2$ (b) $\sqrt{2}$
(c) $1/\sqrt{2}$ (d) $1/3$
41. In a capillary tube experiment, a vertical 30 cm long capillary tube is dipped in water. The water rises upto a height of 10 cm due to capillary action. If the experiment is conducted in a freely falling elevator, the length of the water column becomes
(a) 10 cm (b) 20 cm
(c) 30 cm (d) zero
42. A stone is lying in a fluid stream. The force acting on it depends on the density of the fluid, the velocity of flow and the maximum area of cross-section perpendicular to the direction of flow. The force F and the velocity v of flow are related as
(a) $F \propto \frac{1}{v}$ (b) $F \propto v$
(c) $F \propto v^2$ (d) $F \propto \frac{1}{v^2}$
43. A cubical room is formed with six plane mirrors. An insect moves along the diagonal of the floor with uniform speed. The velocities of its images in two adjacent walls are $20\sqrt{2} \text{ cm s}^{-1}$, then the velocity of the image formed by the roof is
(a) $10\sqrt{2} \text{ cm s}^{-1}$ (b) 20 cm s^{-1}
(c) $20\sqrt{2} \text{ cm s}^{-1}$ (d) 40 cm s^{-1}
44. A bullet is fired from a gun. The force on the bullet is given by $F = 600 - 2 \times 10^5 t$, where F is in newton and t is in seconds. The force on the bullet becomes zero as soon as it leaves the barrel. What is the average impulse imparted to the bullet?
(a) 9 N s (b) zero
(c) 0.9 N s (d) 1.8 N s
45. An object of mass 5 kg falls from rest through a vertical distance of 20 m and acquires a velocity of 10 m s^{-1} . The work done by the push of air on the object is ($g = 10 \text{ m s}^{-2}$)
(a) 250 J (b) -500 J
(c) 750 J (d) -750 J

SOLUTIONS

1. (d): Let speed of the third car be $v \text{ km h}^{-1}$.
Then, relative speed of third car with respect to two cars = $(v + 30) \text{ km h}^{-1}$
Two cars are separated by 5 km and third car crosses them in 4 min.

$$\therefore v + 30 = \frac{5 \times 60}{4} = 75$$

$$\Rightarrow v = 75 - 30 = 45 \text{ km h}^{-1}$$

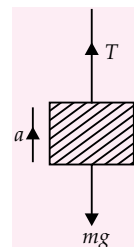
2. (c): Effective upward force, $F = T - mg$
 $= 310 - 24 \times 9.8 = 74.8 \text{ N}$
 \therefore upward acceleration, $a = F/m$
 $a = 74.8/24 = 3.12 \text{ m s}^{-2}$

As $s = ut + \frac{1}{2}at^2$

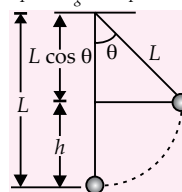
$$4.6 = 0 + \frac{1}{2} \times 3.12 \times t^2$$

$$\text{or } t^2 = \frac{4.6}{1.56} = 2.95$$

$$\text{or } t = \sqrt{2.95} \approx 1.7 \text{ s}$$



3. (c): $W = \Delta K$ or $W_T + W_g + W_F = 0$



Here, W_T = work done by tension = 0

W_g = work done by force of gravity

$$= -mgh = -mgL(1 - \cos\theta)$$

$$\therefore W_F = -W_g = mgL(1 - \cos\theta)$$

4. (b): Let at the time of explosion, velocity of one piece of mass $m/2$ is $(10\hat{i})$. If velocity of other be \vec{v}_2 , then from conservation of momentum (since there is no force in horizontal direction), horizontal component of \vec{v}_2 , must be $-10\hat{i}$.
 \therefore relative velocity of two parts in horizontal direction = 20 m s^{-1} .

Time taken by ball to fall through 45 m,

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 45}{10}} = 3 \text{ s}$$

and time taken by ball to fall through first 20 m,

$$t' = \sqrt{\frac{2h'}{g}} = \sqrt{\frac{2 \times 20}{10}} = 2 \text{ s}.$$

Hence time taken by ball pieces to fall from 25 m height to ground = $t - t' = 3 - 2 = 1 \text{ s}$.

\therefore horizontal distance between the two pieces at the time of striking on ground = $20 \times 1 = 20 \text{ m}$.

5. (c): Energy required to raise a satellite to a height h (i.e. at distance r ($= R + h$) from the centre of earth) is

$$E_1 = -\frac{GMm}{r} - \left(-\frac{GMm}{R} \right)$$

$$= \frac{GMm(r - R)}{rR} = \frac{GMmh}{Rr}$$

Energy spent in putting the satellite in orbit is

$$E_2 = \frac{1}{2}mv_o^2 = \frac{1}{2}m \frac{GM}{r} \quad \left(\because v_o = \sqrt{\frac{GM}{r}} \right)$$

$$\therefore \frac{E_1}{E_2} = \frac{2h}{R}$$

6. (d): Increase in length due to rise in temperature,

$$\Delta L = \alpha L \Delta T$$

$$\text{As, } Y = \frac{FL}{A \Delta L}, \text{ so } F = \frac{YA \Delta L}{L}$$

$$= \frac{YA \times \alpha L \Delta T}{L} = YA \alpha \Delta T$$

$$\therefore F = 2 \times 10^{11} \times 10^{-6} \times 1.1 \times 10^{-5} \times 20 = 44 \text{ N}$$

7. (c): Let h_1, h_2 be the height of mercury and water layers in the vessel. Let A be the area of cross-section of the vessel. Since their weights are equal, so

$$(A \times h_1) \times 13.6 \times g = (A \times h_2) \times 1 \times g$$

$$\text{or } h_2 = 13.6 h_1$$

$$\text{Given, } h_1 + h_2 = 29.2 \text{ cm}$$

$$\text{or } h_1 + 13.6 h_1 = 29.2$$

$$\text{or } h_1 = 2 \text{ cm}$$

$$\therefore h_2 = 13.6 \times 2 = 27.2 \text{ cm}$$

Total pressure exerted by mercury and water on the bottom of vessel

$$= h_1 \times 13.6 \times g + h_2 \times 1 \times g$$

$$= 2 \times 13.6 \times 980 + 27.2 \times 1 \times 980 \text{ dyne}$$

$$= 2 + \frac{27.2}{13.6} = 2 + 2 = 4 \text{ cm of Hg}$$

8. (d): Mean translational K.E. per molecule of a gas
- $$= \frac{3}{2}kT$$

Kinetic energy of an electron when accelerated through a potential difference, $V = eV$

$$\therefore eV = \frac{3}{2}kT$$

$$T = \frac{2eV}{3k} = \frac{2 \times (1.6 \times 10^{-19}) \times 10}{3 \times 1.38 \times 10^{-23}} = 7.73 \times 10^4 \text{ K}$$

9. (c): $\vec{r}_1 = \sqrt{2}\hat{i} + \sqrt{2}\hat{j}$

$$|\vec{r}_1| = r_1 = \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2} = 2$$

$$\vec{r}_2 = 2\hat{i} + 0\hat{j}$$

$$|\vec{r}_2| = r_2 = 2$$

Potential at point A,

$$V_A = \frac{1}{4\pi\epsilon_0} \frac{q}{r_1}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{10^{-3} \times 10^{-6}}{2}$$

Potential at point B,

$$V_B = \frac{1}{4\pi\epsilon_0} \frac{q}{r_2} = \frac{1}{4\pi\epsilon_0} \frac{10^{-3} \times 10^{-6}}{2}$$

$$\therefore V_A - V_B = 0.$$

10. (c): In case of linear rheostat, resistance $R \propto \text{length } L$

$$\therefore \frac{R_{AC}}{R_{AB}} = \frac{AC}{AB}$$

$$\text{Here, } R_{AB} = 12000 \Omega$$

$$\text{and } AC = \frac{1}{4} AB$$

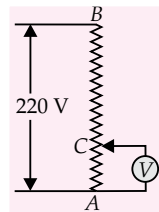
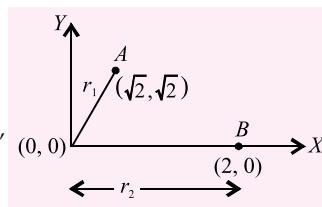
$$\therefore R_{AC} = 12000 \times \frac{1}{4} = 3000 \Omega$$

As the resistance R_{AC} ($= 3000 \Omega$) is in parallel with voltmeter of resistance of 6000Ω . Therefore, the effective resistance between points A and C will be

$$R'_{AC} = \frac{3000 \times 6000}{(3000 + 6000)} = 2000 \Omega$$

Resistance between points B and C

$$R_{BC} = R_{AB} - R_{AC} = 12000 \Omega - 3000 \Omega = 9000 \Omega$$



R_{BC} and R'_{AC} are in series. Therefore, the voltmeter reading will be

$$V_{AC} = \frac{R'_{AC} V_{AB}}{(R_{BC} + R'_{AC})} = \frac{2000}{(9000 + 2000)} \times 220 = 40 \text{ V}$$

11. (c): Here, $l = 50 \text{ cm} = 50 \times 10^{-2} \text{ m}$

$$m = 50 \text{ g} = 50 \times 10^{-3} \text{ kg}, I = 5.0 \text{ A}$$

Tension in the wires is zero, if the force on the rod due to magnetic field is equal and opposite to the weight of the rod.

$$\text{i.e., } mg = BIl$$

$$B = \frac{mg}{Il}$$

Substituting the given values, we get

$$B = \frac{50 \times 10^{-3} \times 10}{5 \times 50 \times 10^{-2}} = 0.2 \text{ T}$$

12. (d): In the process of getting straight up and stand from squatting position, the man exerts a variable force (F) on the ground to set his body in motion. This force is in addition to the force required to support his weight (mg). Once the man is in standing position, F becomes zero.

13. (d): When the centre of mass of the rod rises through h , increase in PE = mgh .

$$\text{Kinetic energy of the rod} = \frac{1}{2} I \omega^2 = \frac{1}{2} \left(\frac{ml^2}{3} \right) \omega^2$$

$$\text{Thus, } mgh = \left(\frac{ml^2}{6} \right) \omega^2 \text{ or } h = \frac{1}{6} \frac{l^2 \omega^2}{g}$$

14. (d): As a given pole (N or S) of suspended magnet goes into the coil and comes out of it, current is induced in the coil in two opposite directions. Therefore, galvanometer deflection goes to left and right both. As amplitude of oscillation of magnet goes on decreasing, so does the amplitude of deflection.

15. (d): Energy stored in capacitor = $\frac{1}{2} CV^2$

$$= \frac{1}{2} \times 16 \times 10^{-6} \times 20^2 = 3.2 \times 10^{-3} \text{ J}$$

$$\text{Energy supplied to inductor} = 3.2 \times 10^{-3} \text{ J}$$

$$\therefore \frac{1}{2} LI^2 = 3.2 \times 10^{-3}$$

$$\text{or } \frac{1}{2} \times (40 \times 10^{-3}) I^2 = 3.2 \times 10^{-3}$$

$$\Rightarrow I^2 = \frac{2 \times 3.2}{40}$$

$$\Rightarrow I = \frac{4}{10} = 0.4 \text{ A}$$

16. (b): For first dark fringe ($n = 1$)

$$x = (2n - 1) \frac{\lambda D}{2d} = \frac{\lambda D}{2d}$$

$$\text{Angular position, } \theta = \frac{x}{D} = \frac{\lambda}{2d}$$

$$= \frac{5460 \times 10^{-10}}{2 \times 10^{-4}} \text{ radian}$$

$$= 2730 \times 10^{-6} \times \frac{180}{\pi} \text{ degree} = 0.16^\circ$$

17. (a): As, $\frac{1}{F} = \frac{2}{f_e} + \frac{1}{f_m}$

For a plane mirror $f_m = \infty$

$$\therefore F = \frac{f_e}{2} = \frac{20}{2} = 10 \text{ cm}$$

Hence arrangement behaves as a convex mirror of focal length 10 cm.

18. (d): Let initial no. of U atoms = N_0

After time t (age of rock), let no. of atoms remaining undecayed = N .

No. of atoms converted into lead = $N_0 - N$

$$\therefore \frac{238N}{226(N_0 - N)} = \frac{4}{3} \Rightarrow \frac{N_0}{N} = 1.79$$

$$\text{Now, } t = \frac{T \log N_0 / N}{\log 2}$$

$$= \frac{4.5 \times 10^9 \times \log 1.79}{0.301} = 3.78 \times 10^9 \text{ years}$$

19. (b): Let h be the depth of bottle in water then

$$A h \rho g = mg \text{ or } h = \frac{m}{A \rho} = \frac{200}{50 \times 1} = 4 \text{ cm}$$

$$\text{Now, } T = 2\pi \sqrt{\frac{h}{g}} \text{ or } v = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{g}{h}}$$

$$= \frac{7}{2 \times 22} \sqrt{\frac{980}{4}} = 2.5 \text{ Hz}$$

20. (a): Given $T = 20 \text{ N}$,

$$A = 0.2 \text{ mm}^2 = 0.2 \times 10^{-6} \text{ m}^2$$

$$\rho = 7.5 \text{ g cm}^{-3} = 7.5 \times 10^3 \text{ kg m}^{-3}$$

Mass per unit length of the wire,

$$\mu = A \rho = 0.2 \times 10^{-6} \times 7.5 \times 10^3 = 1.5 \times 10^{-3} \text{ kg m}^{-1}$$

When a string is plucked at the midpoint it vibrates with fundamental frequency.

Fundamental frequency of the stretched string is

$$v = \frac{1}{2L} \sqrt{\frac{T}{\mu}} = \frac{1}{2L} \sqrt{\frac{20}{1.5 \times 10^{-3}}}$$

Speed of transverse wave, $v = v(2L)$

$$\therefore v = \frac{1}{2L} \sqrt{\frac{20}{1.5 \times 10^{-3}}} \times 2L = 116 \text{ m s}^{-1}$$

21. (d): Here, $C = \frac{\epsilon_0 A}{d} = 9 \text{ pF}$

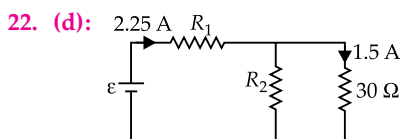
Now, $C_1 = \frac{\epsilon_0 K_1 A}{d/3} = \frac{3K_1 \epsilon_0 A}{d}$
 $= 3 \times 3 \times 9 = 81 \text{ pF}$

$C_2 = \frac{\epsilon_0 K_2 A}{2d/3} = \frac{3K_2 \epsilon_0 A}{2d}$
 $= \frac{3 \times 6}{2} \times 9 = 81 \text{ pF}$

As C_1, C_2 are connected in series,

$\therefore \frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{81} + \frac{1}{81} = \frac{2}{81}$

$\Rightarrow C_s = \frac{81}{2} = 40.5 \text{ pF}$



Current through R_2 is $(2.25 - 1.5) \text{ A} = 0.75 \text{ A}$

Voltage across $30 \Omega = 1.5 \times 30 = 45 \text{ V}$

As R_2 and 30Ω are in parallel

\therefore Voltage across, $R_2 = 45 \text{ V}$

$\therefore R_2 = \frac{45 \text{ V}}{0.75 \text{ A}}$

$R_2 = 60 \Omega$

Also, $R_1 = 60 \Omega$ ($\because R_1 = R_2$ (Given))

Voltage across, $R_1 = 2.25 \times 60 \Omega = 135 \text{ V}$

$\therefore \epsilon = (135 + 45) \text{ V} = 180 \text{ V}$

23. (c): (i) As $\frac{l}{100-l} = \frac{R}{S}$,

$\therefore \frac{R}{S} = \frac{20}{100-20} = \frac{1}{4}$,

which means $R < S$

(ii) As $\frac{l'}{100-l'} = \frac{R+15}{S}$,

$\therefore \frac{40}{100-40} = \frac{R+15}{S} = \frac{2}{3}$

or $\frac{R}{S} + \frac{15}{S} = \frac{2}{3}$ or $\frac{1}{4} + \frac{15}{S} = \frac{2}{3} \Rightarrow S = 36 \Omega$

$\therefore R = \frac{S}{4} = 9 \Omega$

24. (a): For the point O,

$B_1 = \left(\frac{\mu_0 I_c}{2R} \right) \otimes$ (due to circular loop)

$B_2 = \left(\frac{\mu_0 I_l}{2\pi H} \right) \otimes$ (due to long straight wire)

Since \vec{B}_1 and \vec{B}_2 act in opposite direction, for resultant magnetic field to be zero, $B_1 = B_2$ where,

$H = \frac{I_l R}{I_c \pi}$

25. (c): Force on 10 kg block = $ma = 10 \times 12 = 120 \text{ N}$.

Total force applied = 200 N

Force acting on 20 kg block = $200 - 120 = 80 \text{ N}$

\therefore acceleration of 20 kg block = $80/20 = 4 \text{ m s}^{-2}$.

26. (a): As angular momentum is conserved in the absence of a torque, therefore

$I_0 \omega_0 = I \omega$

$\left(\frac{2}{5} MR^2 \right) \left(\frac{2\pi}{T_0} \right) = \left[\frac{2}{5} MR^2 + \frac{2}{5} \frac{MR^2}{5 \times 10^{19}} \right] \frac{2\pi}{T}$

$\frac{T}{T_0} = 1 + \frac{1}{5 \times 10^{19}}$

$\frac{T}{T_0} - 1 = \frac{1}{5 \times 10^{19}} = 2 \times 10^{-20}$

$\frac{T - T_0}{T_0} = 2 \times 10^{-20}$ or $\frac{\Delta T}{T_0} = 2 \times 10^{-20}$

27. (b): Twisting couple per unit twist for solid cylinder,

$\tau_1 = \frac{\pi \eta R^4}{2l}$... (i)

for hollow cylinder, $\tau_2 = \frac{\pi \eta (r_2^4 - r_1^4)}{2l}$

$\therefore \tau_2 = \tau_1 \frac{r_2^4 - r_1^4}{R^4}$ using (i),

$= \frac{0.1 \times (5^4 - 4^4)}{3^4} = \frac{36.9}{81}$

$= 0.455 \text{ N m}$

28. (d): According to Newton's law of cooling,

$\frac{T_1 - T_2}{t} = K \left[\frac{T_1 + T_2}{2} - T_0 \right]$

$\therefore \frac{60 - 50}{10} = K \left[\frac{60 + 50}{2} - 25 \right]$... (i)

Let T be the temperature after another 10 minutes

$\therefore \frac{50 - T}{10} = K \left[\frac{T + 50}{2} - 25 \right]$... (ii)

Dividing (i) by (ii), we get

$\frac{10}{50 - T} = \frac{30 \times 2}{T} \therefore T = 42.85^\circ \text{C}$

29. (a): $P_1 = P$ and $T_1 = T$,

$P_2 = P + \frac{0.4}{100} P = 1.004P$, $T_2 = (T + 1)$

$$\text{As } \frac{P_2}{P_1} = \frac{T_2}{T_1}$$

$$\therefore \frac{1.004P}{P} = \frac{T+1}{T} = 1 + \frac{1}{T}$$

$$\text{or } T = \frac{1}{0.004} = 250 \text{ K}$$

30. (c): Here, $\frac{r_1}{r_2} = \frac{1 \text{ mm}}{2 \text{ mm}} = \frac{1}{2}$. When the spheres

are connected by a conducting wire, then the potentials on both conductors are same

$$\text{i.e., } V_1 = V_2,$$

$$\therefore \frac{q_1}{4\pi\epsilon_0 r_1} = \frac{q_2}{4\pi\epsilon_0 r_2} \Rightarrow \frac{q_1}{q_2} = \frac{r_1}{r_2} = \frac{1}{2}$$

$$\text{Now, } \frac{E_1}{E_2} = \frac{q_1}{q_2} \cdot \left(\frac{r_2}{r_1}\right)^2 = \frac{1}{2} \times \left(\frac{2}{1}\right)^2 = 2:1$$

31. (a): Net resistance of 6Ω , 3Ω and 2Ω is

$$R = \frac{6 \times 3}{6 + 3} + 2 = 4 \Omega$$

As resistance of arms in parallel are same, hence current in upper arm, $I = 1 \text{ A}$

$$\text{At junction, } I = I_1 + I_2 \quad \dots(i)$$

$$\text{Potential difference across } 6 \Omega \text{ and } 3 \Omega = 6I_1 = 3I_2$$

$$\text{or } I_2 = 2I_1 \quad \dots(ii)$$

Solving eqns. (i) and (ii), we get,

$$I_2 = \frac{2}{3} \text{ A}, I_1 = \frac{1}{3} \text{ A}$$

$$\text{Potential difference across } 3 \Omega = \frac{2}{3} \text{ A} \times 3 \Omega = 2 \text{ V}$$

32. (b): Consider a hypothetical ring of radius x and thickness dx of a disc as shown in figure.

Charge on the ring,

$$dq = \frac{q}{\pi r^2} \times (2\pi x dx)$$

Current due to rotation of charge on ring is

$$dI = \frac{dq}{T} = \frac{dq}{1/n} = ndq = \frac{nq2x dx}{r^2}$$

Magnetic field at the centre O due to current on ring element is

$$dB = \frac{\mu_0 dI}{2x} = \frac{\mu_0 nq2x dx}{r^2(2x)} = \frac{\mu_0 nq dx}{r^2}$$

Total magnetic field induction due to current on whole disc is

$$B = \frac{\mu_0 nq}{r^2} \int_0^r dx = \frac{\mu_0 nq}{r^2} (x)_0^r = \frac{\mu_0 nq}{r}$$

33. (d): Magnetic field intensity at a distance r from the straight wire carrying current I is

$$B = \frac{\mu_0 I}{2\pi r}$$

As area of loop, $A = a^2$
and magnetic flux $\phi = BA$

$$\therefore \phi = \frac{\mu_0 I a^2}{2\pi r}$$

The induced emf in the loop is

$$e = \left| \frac{d\phi}{dt} \right| = \left| \frac{d}{dt} \frac{\mu_0 I a^2}{2\pi r} \right|$$

$$e = \frac{\mu_0 I a^2}{2\pi r^2} \frac{dr}{dt} = \frac{\mu_0 I a^2 v}{2\pi r^2}$$

where, $v = \frac{dr}{dt}$ is velocity.

34. (a): As, $\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$

$$\therefore \frac{1}{0.06} = (1.5 - 1) \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

(Taking R_1 positive and R_2 negative)

$$\text{or } \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{0.06 \times 0.5} = \frac{100}{3}$$

According to question, $\frac{R_1}{R_2} = \frac{1}{2}$ or $R_2 = 2R_1$

$$\therefore \frac{3}{2R_1} = \frac{100}{3}$$

$$R_1 = \frac{9}{200} = 0.045 \text{ m}$$

$$R_2 = 2R_1 = 2 \times 0.045 \text{ m} = 0.09 \text{ m}$$

35. (c): $R_D = \frac{V_D^2}{P_D} = \frac{(0.5 \text{ V})^2}{0.1 \text{ W}} = 2.5 \Omega$,

$$I_D = \frac{V_D}{R_D} = 0.2 \text{ A}$$

Total resistance required in the circuit,

$$R_{eq} = \frac{V}{I_D} = \frac{1.5 \text{ V}}{0.2 \text{ A}} = 7.5 \Omega$$

Resistance of the series resistor,

$$R = R_{eq} - R_D = 7.5 \Omega - 2.5 \Omega = 5 \Omega$$

36. (a): For first line of Lyman series of hydrogen atom,

$$\frac{1}{\lambda_1} = R \left(\frac{1}{1^2} - \frac{1}{2^2} \right) = \frac{3R}{4}$$

For second line of Balmer series for a hydrogen-like ion,

$$\frac{1}{\lambda_2} = Z^2 R \left(\frac{1}{2^2} - \frac{1}{4^2} \right) = Z^2 \frac{3R}{16}$$

Since $\lambda_1 = \lambda_2$

$$Z^2 R \left(\frac{3}{16} \right) = R \left(\frac{3}{4} \right) \text{ or } Z^2 = 4, \Rightarrow Z = 2$$

37. (b): Work function of the cathode surface,

$$\phi_0 = 4.2 \text{ eV}$$

Energy of the photon emitted by hydrogen discharge tube,

$$h\nu = 13.6 \text{ eV}$$

$$\text{As } eV_0 = h\nu - \phi_0 = 13.6 \text{ eV} - 4.2 \text{ eV} = 9.4 \text{ eV},$$

$$\therefore V_0 = 9.4 \text{ V}$$

In order to reduce photocurrent to zero, anode must be held negative relative to the cathode.

$$\text{Hence, } V_0 = -9.4 \text{ V}$$

38. (b): Image formed by convex lens acts as a *virtual object* for concave lens, where $u = (30 \text{ cm} - 26 \text{ cm}) = 4 \text{ cm}$.

For concave lens, $f = -20 \text{ cm}$

$$\text{From } \frac{1}{f} = \frac{1}{v} - \frac{1}{u},$$

$$v = 5 \text{ cm}$$

Magnification, $m = v/u = 1.25$

$$\text{Also } m = \frac{I}{O}, I = m \times O = 1.25 \times 2 \text{ cm} = 2.5 \text{ cm}$$

39. (a): Limit of resolution, $d\theta = \frac{1.22\lambda}{D}$

Also, $d\theta = \frac{d}{y}$ where d is the distance between the two dots and y is their distance from the eye.

$$\text{Thus, } y = \frac{Dd}{1.22\lambda}$$

$$\text{Here, } D = 3 \text{ mm} = 3 \times 10^{-3} \text{ m}$$

$$d = 1 \text{ mm} = 10^{-3} \text{ m}$$

$$\lambda = 500 \text{ nm} = 500 \times 10^{-9} \text{ m}$$

$$\therefore y = \frac{Dd}{1.22\lambda} = \frac{3 \times 10^{-3} \times 10^{-3}}{500 \times 10^{-9} \times 1.22} = 5 \text{ m}$$

40. (c): Applying the law of conservation of energy, $\text{PE} + \text{KE} = 0$.

$$\therefore -\frac{GMm}{(R+R)} + \frac{1}{2}m(fv)^2 = 0$$

$$\text{or } fv = \sqrt{\frac{GM}{R}} = \sqrt{gR} = \frac{v}{\sqrt{2}} \quad (\because v = \sqrt{2gR})$$

$$\Rightarrow f = 1/\sqrt{2}$$

41. (c): In a freely falling elevator, $g = 0$.

$$\text{As } h = \frac{2T \cos \theta}{r\rho g}, \text{ so, } h = \infty$$

In such a case, the radius of the meniscus will adjust itself in such a way that there would be no overflowing of water. At this stage, the length of the water column becomes equal to the length of the tube, i.e., 30 cm.

42. (c): Let $F \propto d^a v^b a^c$ or $F = K d^a v^b a^c$ where K is dimensionless constant.

$$[\text{MLT}^{-2}] = [\text{ML}^{-3}]^a (\text{LT}^{-1})^b (\text{L}^2)^c$$

$$= \text{M}^a \text{L}^{-3a+b+2c} \text{T}^{-b}$$

Equating the power of M , L and T , we get

$$\therefore a = 1, -3a + b + 2c = 1, -b = -2$$

Solving, we get

$$a = 1, b = 2, c = 1$$

$$\therefore F \propto v^2$$

43. (b): Velocity of the insect along the diagonal

$$= v \cos 45^\circ = 20\sqrt{2} \cos 45^\circ$$

$$= 20\sqrt{2} \times 1/\sqrt{2} = 20 \text{ cm/s}$$

Therefore, the velocity of the image on the roof

$$= 20 \text{ cm/s}$$

44. (c): As $F = 600 - 2 \times 10^5 t$

$$\therefore \text{At } t = 0, F = 600 \text{ N}$$

As $F = 0$, on leaving the barrel,

$$\therefore 0 = 600 - 2 \times 10^5 t$$

$$t = \frac{600}{2 \times 10^5} = 3 \times 10^{-3} \text{ s}$$

This is the time spent by the bullet in the barrel.

$$\text{Average force} = \frac{600 + 0}{2} = 300 \text{ N.}$$

$$\therefore \text{Average impulse imparted} = F \times t$$

$$= 300 \times 3 \times 10^{-3} = 0.9 \text{ N s}$$

45. (d): If P is upward push of air, then resultant force on the body $= (mg - P)$

From work energy theorem,

$$F \times s = \frac{1}{2}mv^2$$

$$(mg - P) \times s = \frac{1}{2}mv^2$$

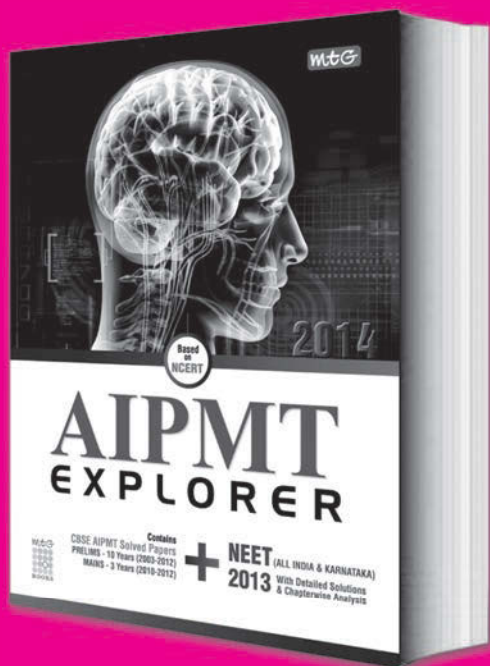
$$(50 - P) \times 20 = \frac{1}{2} \times 5 \times 10^2 = 250$$

$$P = 50 - 12.5 = 37.5 \text{ N}$$

\therefore Work done by this push of air

$$= P_s \cos 180^\circ = 37.5 \times 20(-1) = -750 \text{ J.}$$

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Q1. Explain why, under static conditions, all points on a conductor must be at the same electric potential.

– Punit, Rajasthan

Ans. If two points on a conducting object were at different potentials, then free charges in the object would move, and we would not have static conditions, in contradiction to the initial assumption. (Free positive charges would migrate from higher to lower potential locations; free electrons would move rapidly from lower to higher potential locations.) The charges would continue to move until the potential became equal everywhere in the conductor.

Q2. Does a wire connected to a battery emit an electromagnetic wave?

– Gaurav Sharma, Delhi

Ans. No. The wire will emit electromagnetic waves only if the current varies in time. The radiation is the result of accelerating charges, which can only occur when the current is not constant.

Q3. When looking outdoors through a glass window at night, you sometimes see a double image of yourself. Why?

– Manish, Punjab

Ans. Reflection occurs whenever there is an interface between two different media. For the glass in the window, there are two such surfaces. The first is the inner surface of the glass, and the second is the outer surface of the glass. Each of these interfaces results in an image.

Q4. A baseball is projected into the air at an upward angle to the ground. As it moves through its trajectory, its velocity and therefore its momentum constantly change. Is this a violation of conservation of momentum?

– Rashmi, Bangalore

Ans. The principle of conservation of momentum states that the momentum of an isolated system of particles is conserved. The statement of the question

suggests that the ball is to be considered the system. A ball projected through the air is subject to the gravitational force as an external force, so it is not an isolated system, and we would not expect its momentum to be conserved. If we consider this in a little more detail, the gravitational force is in the vertical direction, so it is actually only the vertical component of the momentum that changes due to this force. In the horizontal direction, no force is exerted (ignoring air friction), so the horizontal component of the momentum is conserved.

If we consider the baseball and the Earth as a system of two particles, the gravitational force is an internal force to this system. The momentum of the ball-Earth system is conserved. The downward force from the thrower's feet as the ball is thrown gives the Earth an initial motion. As the ball rises and falls, the Earth initially sinks and then rises (although imperceptibly!), due to the upward gravitational force from the ball, so that the total momentum of the system remains unchanged.

Q5. How can the total energy of an atom be negative? For that matter, how can any energy be negative?

– Soumya Das, W.B.

Ans. The way the total energy of an atom, and in fact other energies, can be negative is through an arbitrary choice of the zero point for potential energies. The electrostatic potential energy between charged particles is arbitrarily chosen to be zero when the particles are separated by an infinite distance. When two particles having opposite signs, such as the electron and proton in a hydrogen atom, are a finite distance apart, the potential energy is less than zero, or negative. If they are sufficiently close, the magnitude of the negative potential energy may exceed the positive kinetic energy, making the total energy negative.

Q6. You set up two oscillating systems, a simple pendulum and a block hanging from a vertical spring. You carefully adjust the length of the pendulum so that both oscillators have the same period. You now take the two oscillators to the Moon. Will they still have the same period as each other? What happens if you observe the two oscillators in an orbiting space shuttle?

– Santosh, Bihar

Ans. The block hanging from the spring will have the same period on the Moon that it had on the Earth because the period depends on the mass of the block and the force constant of the spring, neither of which have changed. The period of the pendulum on the Moon will be different from its period on the Earth because the period of the pendulum depends on the

value of g . Because g is smaller on the Moon than on the Earth, the pendulum will oscillate with a longer period.

In the orbiting space shuttle, the block-spring system will oscillate with the same period as on the Earth when it is set into motion, because the period does not depend on gravity. The pendulum will not oscillate at all. If you pull it to the side from a direction you define as “vertical” and release it, it stays there. Because the shuttle is in free-fall while in orbit around the Earth, the effective gravity is zero, and there is no restoring force on the pendulum.

Q7. If stereo speakers are connected to the amplifier “out of phase,” one speaker is moving outward when the other is moving inward. This results in a weakness in the bass notes, which can be corrected by reversing the wires on one of the speaker connections. Why are only the bass notes affected in this case, and not the treble notes?

– Pulkit, Haryana

Ans. Imagine that you are sitting in front of the speakers, midway between them. Then, the sound from each speaker travels the same distance to you, so the phase difference in the sound is not due to a path difference. Because the speakers are connected out of phase, the sound waves are half a wavelength out of phase on leaving the speaker and consequently, on arriving at your ear. As a result, the sound for all frequencies cancels, in the simplification model of a zero-size head located exactly on the midpoint between the speakers. If the ideal head were moved off the centerline, an additional phase difference is introduced by the path length difference for the sound from the two speakers. In the case of low-frequency, long-wavelength bass notes, the path length differences are a small fraction of a wavelength, so significant cancellation still occurs. For the high-frequency, short-wavelength treble notes, a small movement of the ideal head results in a much larger fraction of a wavelength in path length difference, or even multiple wavelengths. Thus, the treble notes could be in phase with this head movement. If we now add the fact that the head is not of zero size, and the fact that it has two ears, we can see that complete cancellation is not possible, and, with even small movements of the head, one or both ears will be at or near maxima for the treble notes. The size of the head is much smaller than bass wavelengths, however, so the bass notes are significantly weakened over much of the region in front of the speakers.

Q8. Suppose a current-carrying wire has a cross-sectional area that gradually becomes smaller along the wire, so

that the wire has the shape of a very long cone. How does the drift speed of electrons vary along the wire?

– Sachin, Chennai

Ans. Every portion of the wire is carrying the same amount of current—otherwise, charge would build up or disappear somewhere along the wire. Thus for equation $I = \frac{\Delta Q}{\Delta t} = nqv_dA$ to be satisfied, as the cross-sectional area decreases, the drift speed must increase to maintain the constant current. This increased drift speed is a result of the electric field lines in the wire being distributed over a smaller area, thus increasing the magnitude of the field, and, in turn, increasing the electric force on the electrons.



DO YOU KNOW?

- Diamonds can be made from graphite by applying a temperature of 3000°C and pressure as high as 100,000 atm.
- 50 times as much as all the water that is present in the rivers and lakes combined is present in the ground soil.
- The amount of heat in the Earth's entire atmosphere is equivalent to the heat present in the first ten feet of the ocean.
- One of the most amazing and interesting fact about Physics on earth is “The Dead Sea”, which is known for its density due to the presence of salt, as a result of which you can easily float on it without drowning, so one can always claim to be a swimmer there.
- The flashing lightning bolt is 3 times hotter compared to the sun, seems like the sun also has to face competition.
- A rubber band shrinks when heated and expands when cooled because of the change in its Entropy state.
- Due to the effect of Thermal Expansion, the Eiffel Tower is upto 15 cm taller in summer.
- If an atom were the size of a stadium, its electrons would be as small as bees.
- Light does not age.
- At 25, Physicist Lawrence Bragg is the youngest person to receive a Nobel Prize.
- Atom is over 99.9% empty space.
- The effect of relativity made Astronaut Sergel Avdeyev a fraction of a second younger upon his return to Earth after 747 days in space.
- The world's densest wood, the Black Ironwood (*Olea laurifolia*), does not float on water and therefore sinks.
- The mass of our entire atmosphere is estimated to be some 5.5 quadrillion tons (55 followed by 14 zeros).
- The diameter of a proton is approximately 0.000000000001 mm.

Thought Provoking

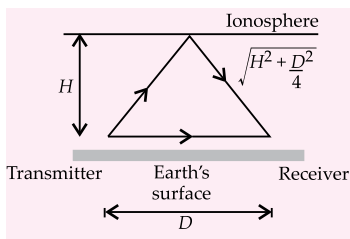
S.H.M. and Wave Motion



Problems

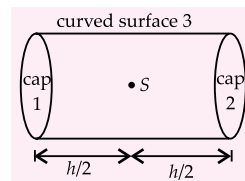
- A train approaching a hill at a speed of 40 km h^{-1} sounds a whistle of frequency 580 Hz when it is at a distance of 1 km from hill. A wind with speed 40 km h^{-1} is blowing in the direction of motion of the train. Find
 - the frequency of the whistle as heard by an observer on the hill,
 - the distance from the hill at which the echo from the hill is heard by the driver and its frequency.
 (Velocity of sound in air = 1200 km h^{-1})

- A short wave radio receiver receives simultaneously two signals from a transmitter 500 km away, one by a path along the surface of the earth and one by reflection from a portion of the ionospheric layer situated at a height of 200 km . The layer acts as a perfect horizontal reflector. When the frequency of the transmitted wave is 10 MHz , it is observed that the combined signal strength varies from maximum to minimum and back to maximum 8 times per minute. With what slow vertical speed is the ionospheric layer moving? (Assume the earth is flat and ignore atmospheric disturbances)

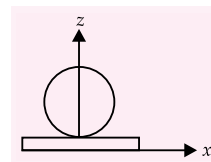


- A point isotropic source with sonic power $P = 0.10 \text{ W}$ is located at the centre of a round hollow cylinder with radius $R = 1 \text{ m}$ and height $h = 2 \text{ m}$.

Assuming sound to be completely absorbed by the walls of the cylinder, find the mean energy of flow rate reaching the lateral curved surface of the cylinder.

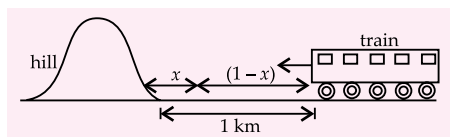


- A rope of mass m and length L is suspended vertically. If a mass M is suspended from the bottom of the rope, find the time for a transverse wave to travel the length of rope.
- A cylinder of mass M and radius R is kept on a rough horizontal platform at one extreme of the platform at $t = 0$. Axis of the cylinder is parallel to y -axis. The platform is oscillating in the x - y plane and displacement from the origin is represented as $x = 2\cos(4\pi t) \text{ m}$. There is no slipping between the cylinder and the platform. Find the acceleration of the centre of mass and the cylinder as a function of time.
- A bead is constrained to move on a smooth circular wire frame in the vertical plane. The frame rotates uniformly with angular velocity about a vertical axis passing through its diameter. In the equilibrium position (relative to frame) the radius drawn to the bead makes an angle ϕ with the vertical. Find the time period of small oscillations about this position.



SOLUTIONS

1. (a) $v_o = v_s \frac{(v \pm v_m \pm v_o)}{(v \pm v_m \mp v_s)}$



The observer is at rest on the hill, i.e., $v_o = 0$
The wind is blowing from source to observer and source is moving towards the observer.

$$\therefore v_o = v_s \left(\frac{v + v_m}{v + v_m - v_s} \right) = 580 \left(\frac{1200 + 40}{1200 + 40 - 40} \right) = 599.3 \text{ Hz}$$

(b) Let x be the distance from the hill at which echo is heard by the train driver.

The distance travelled by the train is $(1 - x)$.

Time taken by train to travel = $\frac{1 - x}{40}$... (i)

Also time taken by sound to move up to the hill and return back to the driver

$$= \frac{1}{1240} + \frac{x}{1160} \quad \dots (ii)$$

Solving (i) and (ii), we get,

$$\frac{1 - x}{40} = \frac{1}{1240} + \frac{x}{1160}$$

$$\Rightarrow 1 - x = \frac{1}{31} + \frac{x}{29}$$

$$\therefore x = \frac{29}{31} \text{ km}$$

When the reflected sound wave travels back to the driver, the wavefronts are moving against the wind. The train driver is listener in motion, thus the frequency heard by listener,

$$v_o = v_s \left(\frac{v - v_m + v_o}{v - v_m} \right) = 599.3 \left(\frac{1200 - 40 + 40}{1200 - 40} \right)$$

$$= 620 \text{ Hz}$$

$$\Rightarrow v_o = 620 \text{ Hz}$$

2. The path difference between direct path on earth's surface D and height of ionospheric layer H is given by

$$\Delta x = 2 \left(H^2 + \frac{D^2}{4} \right)^{1/2} - D \quad \dots (i)$$

The frequency of observed fluctuation is

$$f = \frac{1}{\lambda} \frac{d(\Delta x)}{dt} = \frac{v}{c} \frac{d(\Delta x)}{dt} \quad \dots (ii)$$

where v is frequency of radiation and c is the speed of light.

Differentiating equation (i) with respect to time gives

$$\frac{d(\Delta x)}{dt} = 2H \left(H^2 + \frac{D^2}{4} \right)^{-1/2} \frac{dH}{dt} \quad \dots (iii)$$

Using equation (iii) in equation (ii), we get

$$f = \frac{2vHv}{c} \left(H^2 + \frac{D^2}{4} \right)^{-1/2},$$

where $\frac{dH}{dt} = v$ is the vertical velocity of the layer.

$$\Rightarrow v = \frac{fc}{2Hv} \left(H^2 + \frac{D^2}{4} \right)^{1/2}$$

Here, $f = 8 \text{ minute}^{-1} = \frac{8}{60} \text{ s}^{-1}$

$$C = 3 \times 10^8 \text{ m s}^{-1}, H = 200 \text{ km} = 2 \times 10^5 \text{ m}$$

$$v = 10 \text{ MHz} = 10^7 \text{ Hz}, D = 500 \text{ km} = 5 \times 10^5 \text{ m}$$

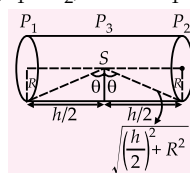
$$\therefore v = \frac{(8/60)(3 \times 10^8)}{2 \times 2 \times 10^5 \times 10^7} \left[(2 \times 10^5)^2 + \frac{(5 \times 10^5)^2}{4} \right]^{1/2}$$

$$v = 3.2 \text{ m s}^{-1}$$

3. Total power, $P = P_1 + P_2 + P_3$

As the end caps are placed symmetrically relative to the source, $P_1 = P_2$.

$$\text{Thus, } P_3 = P - (P_1 + P_2) = P - 2P_1$$



From figure, power flow through ring

$$P_1 = \frac{P}{4\pi} \times [2\pi(1 - \cos\theta)]$$

$$= \frac{P}{2}(1 - \cos\theta) = \frac{P}{2} \left[1 - \frac{h/2}{\sqrt{(h/2)^2 + R^2}} \right]$$

$$= \frac{P}{2} \left[1 - \frac{h}{\sqrt{h^2 + 4R^2}} \right]$$

$$\therefore P_3 = P - 2 \times \frac{P}{2} \left[1 - \frac{h}{\sqrt{h^2 + 4R^2}} \right] = \frac{Ph}{\sqrt{h^2 + 4R^2}}$$

$$= \frac{0.1 \times 2}{\sqrt{(2)^2 + 4(1)^2}}$$

$$\Rightarrow P_3 = 0.07 \text{ W}$$

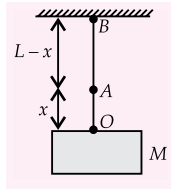
4. Due to the mass of the string, the tension in the string is variable.

\therefore wave velocity is variable.

Consider a point at a distance x from O .

Tension at A,

$$T_A = \left(M + \frac{m}{L}x \right) g$$



where $\frac{m}{L}$ is the mass per unit length of string.

Velocity of wave at point A,

$$v_A = \frac{dx}{dt} = \sqrt{\frac{[M + (m/L)x]g}{(m/L)}}$$

$$\begin{aligned} \therefore \int_0^L \frac{dx}{\sqrt{[M + (m/L)x]g}} &= \int_0^t \sqrt{\frac{L}{m}} dt \\ \Rightarrow 2 \left(\frac{L}{m\sqrt{g}} \right) \left[\sqrt{M + \frac{m}{L}x} \right]_0^L &= \sqrt{\frac{L}{m}} \times t \\ \text{or } t &= 2\sqrt{\frac{L}{mg}} [\sqrt{M+m} - \sqrt{M}] \end{aligned}$$

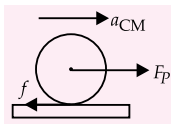
When $M = 0$, $t = 2\sqrt{\frac{L}{g}}$

For $m \ll M$, $t = \sqrt{\frac{mL}{Mg}}$

5. Given equation $x = 2\cos(4\pi t)$ can be equated with $x = A\cos\omega t$, so $\omega = 4\pi \text{ rad s}^{-1}$

$$a = -\omega^2 x = -32\pi^2 \cos(4\pi t) \text{ m s}^{-2}$$

Pseudo force acting on the cylinder = Ma



$$\therefore F = Ma = 32\pi^2 M \cos(4\pi t).$$

If a_{CM} = acceleration of center of mass of the cylinder relative to plank,

From Newton's second law,

$$32\pi^2 M \cos(4\pi t) - f = Ma_{CM} \quad \dots(i)$$

Taking moment about the centre of mass of the cylinder

$$f \cdot R = \left(\frac{1}{2} MR^2 \right) \alpha \quad \dots(ii)$$

$$\text{For pure rolling, } a_{CM} = R\alpha \quad \dots(iii)$$

Hence, solving above three equations, we get

$$\text{Friction, } f = \frac{32}{3} \pi^2 \cdot M \cos(4\pi t) \text{ N}$$

and acceleration of centre of mass of the cylinder

$$\text{relative to platform } a_{CM} = \frac{64}{3} \pi^2 \cos(4\pi t) \text{ m s}^{-2}$$

Acceleration of centre of mass of the cylinder relative to ground

$$\begin{aligned} &= a_{CM} - a = \left(\frac{64}{3} - 32 \right) \pi^2 \cos(4\pi t) \text{ m s}^{-2} \\ &= \frac{-32}{3} \pi^2 \cos(4\pi t) \text{ m s}^{-2} \end{aligned}$$

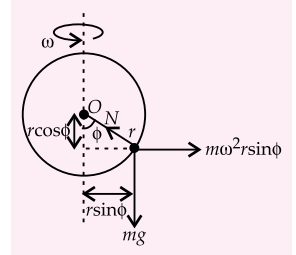
6. Taking moment about centre O of the wire frame.

$$(m\omega^2 r \sin\phi) \times r \cos\phi = mg(r \sin\phi)$$

$$\Rightarrow \omega^2 r \cos\phi = g \quad \dots(i)$$

If bead is further displaced through an angle θ , so that the angle of radius with the vertical becomes $(\theta + \phi)$.

Therefore restoring tangential force is



$$\begin{aligned} mr \frac{d^2}{dt^2} (\phi + \theta) &= -[mg \sin(\phi + \theta) \\ &\quad - m\omega^2 r \sin(\phi + \theta) \cos(\phi + \theta)] \end{aligned}$$

$$\begin{aligned} \text{or } r \frac{d^2\theta}{dt^2} + g \sin\phi \cos\theta + g \cos\phi \sin\theta \\ - \frac{\omega^2 r}{2} \sin 2\phi \cos 2\theta - \frac{\omega^2 r}{2} \cos 2\phi \sin 2\theta &= 0 \quad \dots(ii) \end{aligned}$$

For small θ , $\sin\theta \approx \theta$ and $\cos\theta \approx 1$

Hence,

$$\begin{aligned} r \frac{d^2\theta}{dt^2} + (g \cos\phi - \omega^2 r \cos 2\phi) \theta + \sin\phi (g - \omega^2 r \cos\phi) &= 0 \\ \dots(iii) \end{aligned}$$

From equation (i), $g - \omega^2 r \cos\phi = 0$

$$\Rightarrow g = \omega^2 r \cos\phi$$

$$\begin{aligned} \text{or, } g \cos\phi - \omega^2 r \cos 2\phi &= \omega^2 r \cos^2\phi - \omega^2 r \cos 2\phi \\ &= \omega^2 r \cos^2\phi - \omega^2 r [2\cos^2\phi - 1] \\ &= \omega^2 r \sin^2\phi \quad \dots(iv) \end{aligned}$$

From equation (ii) and (iv)

$$r \frac{d^2\theta}{dt^2} + (\omega^2 r \sin^2\phi) \cdot \theta + 0 = 0$$

$$\Rightarrow \frac{d^2\theta}{dt^2} + \left(\frac{g \sin^2\phi}{r \cos\phi} \right) \theta = 0 \text{ gives } \omega = \sqrt{\frac{g \sin^2\phi}{r \cos\phi}}$$

$$\text{Hence, } T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{r \cos\phi}{g \sin^2\phi}}$$



PHYSICS MUSING

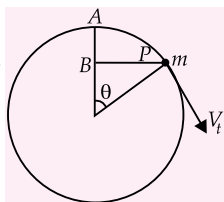
SOLUTION SET-7

1. (d) : Let frame of reference be the one attached to the sphere. With zero velocity of the particle relative to sphere at point A and center of the sphere as the reference point for potential energy, the total mechanical energy of the system at A is

$$E_i = mgR$$

The total mechanical energy at the final position (point P) where the particle makes an angle θ with the vertical is

$$E_f = \frac{1}{2}mv_t^2 + mgR\cos\theta$$



Since the reference frame attached to the sphere accelerates, there is a pseudo force ma opposite to the direction of a . The work done by this force is

$$W = maR\sin\theta = mgR\sin\theta \quad (\because g = a)$$

By the principle of conservation of mechanical energy

$$W = \Delta K + \Delta U$$

$$\Rightarrow mgR\sin\theta = \frac{1}{2}mv_t^2 - mgR(1 - \cos\theta)$$

$$\Rightarrow v_t = [2R(g\sin\theta + g - g\cos\theta)]^{\frac{1}{2}}$$

2. (d): As kinetic energy, $K \propto P^2$

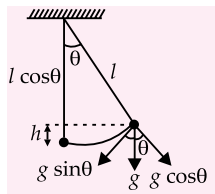
Here the momentum (P) of the body is increased by 20%

$$P' = P + \frac{20}{100}P = \frac{6P}{5}$$

$$\therefore K' = P'^2 = \left(\frac{6}{5}P\right)^2 = \frac{36}{25}P^2 = \frac{36}{25}K$$

$$\frac{K' - K}{K} \times 100 = \frac{11}{25} \times 100 = 44.0\%$$

3. (b):



Suppose θ is the required angle.

At extreme position the velocity of the ball is zero,

thus normal acceleration $a_n = \frac{v_e^2}{l} = 0$,

and tangential acceleration $a_t = g \sin\theta$.

\therefore Total acceleration at extreme position $a_e = g \sin\theta$

At mean position

The velocity of the ball

$$v_m = \sqrt{2gh} = \sqrt{2g(l - l\cos\theta)}$$

and displacements of ball, $h = (l - l\cos\theta)$

The normal acceleration at this position, $a_n = \frac{v_m^2}{l}$

$$= \frac{2g(l - l\cos\theta)}{l} = 2g(1 - \cos\theta)$$

and tangential acceleration $a_t = g \sin\theta = g \sin 0 = 0$

Thus total acceleration at mean position

$$a_m = 2g(1 - \cos\theta)$$

According to given condition, we have

$$a_e = a_m \Rightarrow g \sin\theta = 2g(1 - \cos\theta)$$

$$\Rightarrow 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2} = 2 \left(2 \sin^2 \frac{\theta}{2} \right)$$

$$\text{or } \tan \frac{\theta}{2} = \frac{1}{2} \Rightarrow \theta = 2 \tan^{-1} \left(\frac{1}{2} \right)$$

4. (b): Let θ is the required angle. The velocity of the body at angular position θ is

$$v^2 = 0 + 2g(l \cos\theta) \quad \dots (i)$$

The normal acceleration of bob,

$$a_n = \frac{v^2}{l} = \frac{2gl \cos\theta}{l} = 2g \cos\theta$$

And tangential acceleration of bob, $a_t = g \sin\theta$

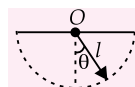
As the resultant acceleration is directed

horizontally so $\tan\theta = \frac{a_n}{a_t}$

$$\text{or } \frac{\sin\theta}{\cos\theta} = \frac{2g \cos\theta}{g \sin\theta}$$

$$\text{or } 2 \cos^2\theta = \sin^2\theta = (1 - \cos^2\theta)$$

$$\Rightarrow \cos^2\theta = \frac{1}{3} \text{ and } \theta = \cos^{-1} \left(\frac{1}{\sqrt{3}} \right)$$



5. (a): The velocity of bob at its lowest position,

$$v^2 = 0 + 2g(l - l\cos 60^\circ) = 2g \times \frac{l}{2}$$

$$\text{or } v = \sqrt{gl} = \sqrt{g \times 1} \quad [\because l = 1 \text{ m}] \quad \dots (i)$$

Let d is the distance of nail from the point of suspension. The bob will have to complete the circle of radius $r = 1 - d$.

For the bob to just perform the revolutions about nail, the minimum speed at the lowest position must be

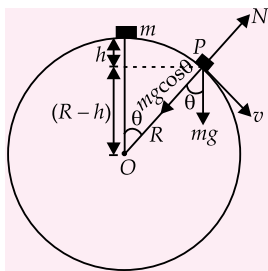
$$\sqrt{5gr} = \sqrt{5g(1-d)} \quad \dots (ii)$$

Equating equations (i) and (ii), we get

$$\sqrt{g} = \sqrt{5g(1-d)}$$

$$\Rightarrow d = \frac{4}{5} = 0.80 \text{ m}$$

6. (b):



Let the particle lose the contact at point P. At point P normal reaction by surface becomes zero. If v is the velocity of the particle at P, then we have

$$v^2 = 0 + 2gh$$

$$\text{or } v = \sqrt{2gh} \quad \dots (i)$$

By Newton's second law, we have

$$mg \cos \theta - N = \frac{mv^2}{R}$$

At P, $N = 0$,

$$\therefore mg \cos \theta = \frac{mv^2}{R} = \frac{m(2gh)}{R} \quad (\text{using (i)})$$

$$\Rightarrow \cos \theta = \frac{2h}{R}$$

$$\text{Therefore, } \frac{2h}{R} = \frac{R-h}{R} \text{ or } h = \frac{R}{3}$$

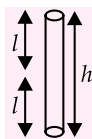
$$\text{Also in figure, } \cos \theta = \frac{R-h}{h}$$

$$\text{Here } \cos \theta = \frac{2}{3} \Rightarrow \theta = \cos^{-1}\left(\frac{2}{3}\right).$$

7. (a, d): As rod is a rigid part of the system, it can take compression ($T < 0$), so the velocity of block at its highest position can be zero to just cross this position.

Let block is given a velocity v at its lowest position, then by third equation of motion, we have

$$v_H^2 = v_L^2 - 2gh$$



$$\text{or } 0 = v^2 - 2g \times 2l$$

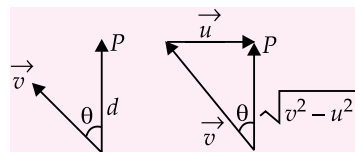
$$\Rightarrow v = \sqrt{4gl} = 2\sqrt{gl}$$

8. (b,c,d)

$$9. (a,c,d) : t = \frac{d}{v \cos \theta}$$

$$t_{\min} = \frac{d}{v}$$

$$t = \frac{d}{\sqrt{v^2 - u^2}}$$



10. (a,b,c,d) : There is no acceleration in x-direction so x component of velocity is constant.

Now the given equation of motion

$$y = ax - bx^2 \quad \dots (i)$$

The standard equation of projectile motion

$$y = (\tan \theta)x - \frac{gx^2}{2u^2 \cos^2 \theta} \quad \dots (ii)$$

Comparing equations (i) and (ii), we get

$$\tan \theta = a \quad \dots (iii)$$

$$\therefore \theta = \tan^{-1}(a)$$

$$\text{and } \frac{g}{2(u \cos \theta)^2} = b \text{ or } (u \cos \theta)^2 = \frac{g}{2b}$$

$$u \cos \theta = \sqrt{\frac{g}{2b}} \Rightarrow u = \sec \theta \sqrt{g/2b}$$

$$v_y (\text{at origin}) = u \sin \theta$$

$$\text{or } = \sec \theta \sqrt{\frac{g}{2b}} \sin \theta$$

$$= \frac{\sin \theta}{\cos \theta} \sqrt{\frac{g}{2b}} = \tan \theta \sqrt{\frac{g}{2b}} = a \sqrt{\frac{g}{2b}} \quad (\text{using (iii)})$$

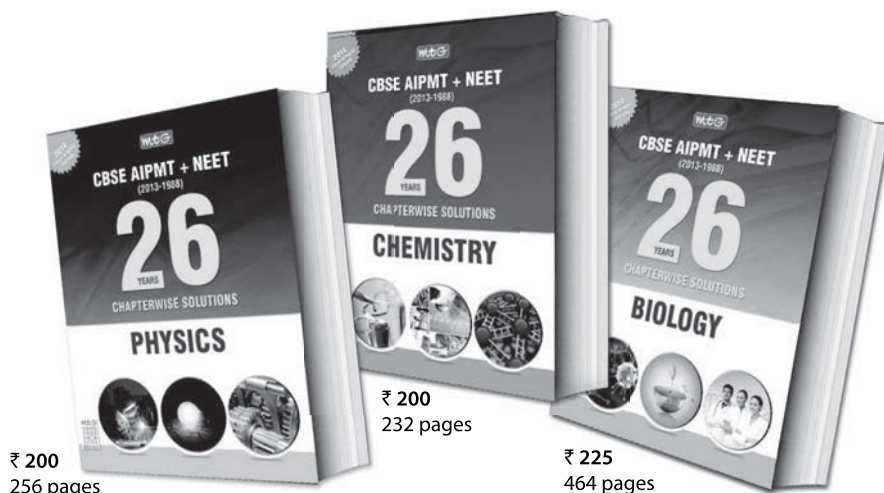
The given equation of motion shows the motion of particle is a projectile motion.

Solution Senders of Physics Musing

SET-7

1. Ayaz Ahmed (Jharkhand)
2. Jenish Jain
3. Shivdatt Rawani
4. Alok Verma
5. Jatin Gupta

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HIGHLIGHTS:

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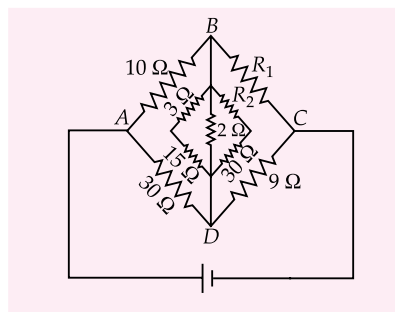


TARGET PMTs

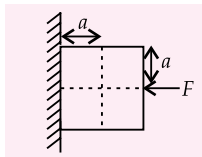
PRACTICE QUESTIONS

Useful for All National and State Level PMTs

- An open organ pipe is closed suddenly with the result that the second overtone of the closed pipe is found to be higher in frequency by 100 than the first overtone of the original pipe. Then the fundamental frequency of the open pipe is
(a) 200 s^{-1} (b) 100 s^{-1}
(c) 300 s^{-1} (d) 250 s^{-1}
- Three unequal resistors in parallel are equivalent to a resistance 1 ohm. If two of them are in the ratio 1 : 2 and if no resistance value is fractional, the largest of the three resistances in ohm is
(a) 4 (b) 6 (c) 5 (d) 12
- From the top of tower, a stone is thrown up. It reaches the ground in t_1 s. A second stone thrown down with the same speed reaches the ground in t_2 s. A third stone released from rest reaches the ground in t_3 s. Then
(a) $t_3 = \frac{(t_1 + t_2)}{2}$ (b) $t_3 = \sqrt{t_1 t_2}$
(c) $\frac{1}{t_3} = \frac{1}{t_1} + \frac{1}{t_2}$ (d) $t_3^2 = t_2^2 - t_1^2$
- A planet is at an average distance d from the sun, and its average surface temperature is T . Assume that the planet receives energy only from the sun, and loses energy only through radiation from its surface. Neglect atmospheric effects. If $T \propto d^{-n}$, the value of n is
(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{4}$
- A glass tube of length 1.0 m is completely filled with water. A vibrating tuning fork of frequency 500 Hz is kept over the mouth of the tube and the water is drained out slowly at the bottom of the tube. If velocity of sound in air is 330 m s^{-1} , then the total number of resonances that occur will be
(a) 2 (b) 3 (c) 1 (d) 5
- In the Wheatstone bridge shown below, in order to balance the bridge, we must have



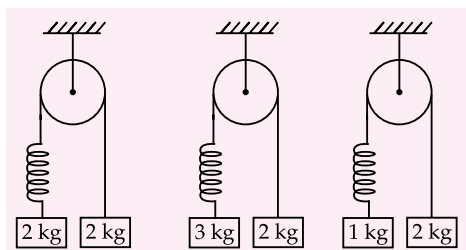
- (a) $R_1 = 3 \Omega$; $R_2 = 3 \Omega$
(b) $R_1 = 6 \Omega$; $R_2 = 15 \Omega$
(c) $R_1 = 1.5 \Omega$; $R_2 = \text{any finite value}$
(d) $R_1 = 3 \Omega$; $R_2 = \text{any finite value}$
- A block of mass m is at rest under the action of force F against a wall as shown in the figure. Which of the following statements is incorrect?
(a) $f = mg$ [where f is the friction force]
(b) $F = N$ [where N is the normal force]
(c) F will not produce torque
(d) N will not produce torque
- In an astronomical telescope in normal adjustment, a straight black line of length L is drawn on the objective lens. The eyepiece forms a real image of this line. The length of this image is l . The magnification of the telescope is
(a) $\frac{L}{l}$ (b) $\frac{L}{l} + 1$
(c) $\frac{L}{l} - 1$ (d) $\frac{L+1}{L-1}$
- A copper wire of resistance 10Ω is in the form of a perfect circle. Two points A and B on it are connected to a battery of emf 5 V and internal



resistance $0.5\ \Omega$. The two segments of the circle have lengths in the ratio $2 : 3$. The net magnetic induction at the center of the circle is

- (a) π (b) zero (c) 2π (d) $\mu_0/4\pi$

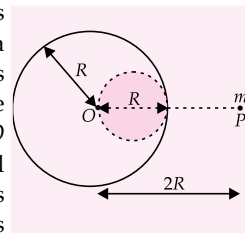
10. A galvanometer of resistance $50\ \Omega$ is connected to a battery of 3 V along with a resistance of $2950\ \Omega$ in series. A full scale deflection of 30 divisions is obtained in the galvanometer. In order to reduce this deflection to 20 divisions, the resistance in series should be
- (a) $6050\ \Omega$ (b) $4450\ \Omega$
(c) $5050\ \Omega$ (d) $5550\ \Omega$
11. What is the value of inductance L for which the current is maximum in a series LCR circuit with $C = 10\ \mu\text{F}$ and $\omega = 1000\text{ s}^{-1}$?
- (a) 1 mH (b) 10 mH
(c) 100 mH (d) cannot be calculated unless R is known
12. A rope of negligible mass is wound around a hollow cylinder of mass 3 kg and radius 40 cm . What is the angular acceleration of the cylinder, if the rope is pulled with a force of 30 N ? Assume that there is no slipping.
- (a) 10 rad s^{-2} (b) 15 rad s^{-2}
(c) 20 rad s^{-2} (d) 25 rad s^{-2}
13. Same spring is attached with 2 kg , 3 kg and 1 kg blocks in three different cases as shown in figure. If x_1 , x_2 , x_3 be the extensions in the spring in the three cases, then



- (a) $x_1 = 0, x_3 > x_2$ (b) $x_1 > x_2 > x_3$
(c) $x_3 > x_2 > x_1$ (d) $x_2 > x_1 > x_3$
14. Two containers of equal volume contain the same gas at pressures P_1 and P_2 and absolute temperatures T_1 and T_2 respectively. On joining the vessels, the gas reaches a common pressure P and a common temperature T . The ratio $\frac{P}{T}$ is
- (a) $\frac{P_1}{T_1} + \frac{P_2}{T_2}$ (b) $\frac{1}{2} \left[\frac{P_1}{T_1} + \frac{P_2}{T_2} \right]$
(c) $\frac{P_1 T_2 + P_2 T_1}{T_1 + T_2}$ (d) $\frac{P_1 T_2 - P_2 T_1}{T_1 - T_2}$

15. The refractive index of the material of a prism is $\sqrt{2}$, and its refracting angle is 30° . One of the refracting surfaces of the prism is made a mirror inwards. A beam of monochromatic light entering the prism from the other face retraces its path, after reflection from mirrored surface, if its angle of incidence on prism is
- (a) 0° (b) 30° (c) 45° (d) 60°
16. A photocell employs photoelectric effect to convert
- (a) change in the frequency of light into a change in the electric current
(b) change in the frequency of light into a change in electric voltage
(c) change in the intensity of illumination into a change in photoelectric current
(d) change in the intensity of illumination into a change in the work function of the photocathode.
17. Ionization potential of hydrogen atom is 13.6 eV . Hydrogen atoms in the ground state are excited by monochromatic radiation of photon energy 12.1 eV . According to Bohr's theory, the spectral lines emitted by hydrogen will be
- (a) one (b) two (c) three (d) four
18. Dimensions of ohm are same as (where h is Planck's constant and e is charge)
- (a) $\frac{h}{e}$ (b) $\frac{h^2}{e}$ (c) $\frac{h}{e^2}$ (d) $\frac{h^2}{e^2}$
19. There is some change in length when a 33000 N tensile force is applied on a steel rod of area of cross-section 10^{-3} m^2 . The change in temperature required to produce the same elongation if the steel rod is heated is
- (The modulus of elasticity is $3 \times 10^{11}\text{ N m}^{-2}$ and coefficient of linear expansion of steel is $1.1 \times 10^{-5}\text{ }^\circ\text{C}^{-1}$)
- (a) 20°C (b) 15°C (c) 10°C (d) 0°C

20. A uniform sphere of mass M and radius R exerts a force F on a small mass m situated at a distance of $2R$ from the centre O of the sphere. A spherical portion of diameter R is cut from the sphere as shown in figure.



The force of attraction between the remaining part of the sphere and the mass m will be

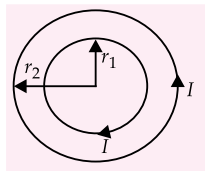
- (a) $\frac{F}{3}$ (b) $\frac{2F}{3}$ (c) $\frac{4F}{3}$ (d) $\frac{7F}{9}$

21. The activity of a radioactive sample is measured as N_0 counts per minute at $t = 0$ and N_0/e counts per minute at $t = 5$ minutes. The time (in minutes) at which the activity reduces to half its value is

(a) $\log_e \frac{2}{5}$ (b) $\frac{5}{\log_e 2}$
(c) $5 \log_{10} 2$ (d) $5 \log_e 2$

22. Two circular concentric loops of radii $r_1 = 20$ cm and $r_2 = 30$ cm are placed in the XY plane as shown in the figure. A current $I = 7$ A is flowing through them. The magnetic moment of this loop system is

(a) $+0.4 \hat{k} \text{ A m}^2$
(b) $-1.5 \hat{k} \text{ A m}^2$
(c) $+1.1 \hat{k} \text{ A m}^2$
(d) $+1.3 \hat{j} \text{ A m}^2$



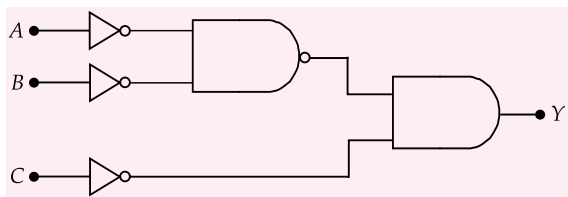
23. A mass of 4 kg suspended from a spring of force constant 800 N m^{-1} executes simple harmonic oscillations. If the total energy of the oscillator is 4 J, the maximum acceleration (in m s^{-2}) of the mass is

(a) 5 (b) 15 (c) 45 (d) 20

24. A body travels a distance of 20 m in the 7th second and 24 m in 9th second. How much distance shall it travel in the 15th second?

(a) 10 m (b) 16 m (c) 24 m (d) 36 m

25. The output Y of the logic circuit as shown in figure is



(a) $(A + B) \bar{C}$ (b) $(A + C) \bar{B}$
(c) $(B + C) \bar{A}$ (d) $A + B + C$

26. A train moves towards a stationary observer with speed 34 m s^{-1} . The train sounds a whistle and its frequency registered by the observer is ν_1 . If the train's speed is reduced to 17 m s^{-1} , the frequency registered is ν_2 . If the speed of sound is 340 m s^{-1} ,

then the ratio $\frac{\nu_1}{\nu_2}$ is

(a) $\frac{18}{19}$ (b) $\frac{1}{2}$ (c) 2 (d) $\frac{19}{18}$

27. A square card of side length 1 mm is being seen through a magnifying lens of focal length 10 cm. The card is placed at a distance of 9 cm from the

lens. The apparent area of the card through the lens is

(a) 1 cm^2 (b) 0.81 cm^2
(c) 0.27 cm^2 (d) 0.60 cm^2

28. Two cylinders of equal size are filled with equal amount of ideal diatomic gas at room temperature. Both the cylinders are fitted with pistons. In cylinder A the piston is free to move, while in cylinder B the piston is fixed. When same amount of heat is supplied to both the cylinders, the temperature of the gas in cylinder A raises by 30 K. What will be the rise in temperature of the gas in cylinder B?

(a) 42 K (b) 30 K
(c) 20 K (d) 56 K

29. Water is in streamline flow along a horizontal pipe with nonuniform cross-section. At a point in the pipe where the area of cross-section is 10 cm^2 , the velocity of water is 1 m s^{-1} and the pressure is 2000 Pa. The pressure at another point where the cross-sectional area is 5 cm^2 is

(a) 4000 Pa (b) 2000 Pa
(c) 1000 Pa (d) 500 Pa

30. A body of mass 3 kg is under a constant force which causes a displacement S in metres in it, given by the relation $S = \frac{1}{3} t^2$ where t is in seconds. Work done by the force in 2 seconds is

(a) $\frac{19}{5} \text{ J}$ (b) $\frac{5}{19} \text{ J}$ (c) $\frac{3}{8} \text{ J}$ (d) $\frac{8}{3} \text{ J}$

31. A solenoid has core of a material with relative permeability 500 and its windings carry a current of 1 A. The number of turns of the solenoid is 500 per metre. The magnetization of the material is nearly

(a) $2.5 \times 10^3 \text{ A m}^{-1}$ (b) $2.5 \times 10^5 \text{ A m}^{-1}$
(c) $2.0 \times 10^3 \text{ A m}^{-1}$ (d) $2.0 \times 10^5 \text{ A m}^{-1}$

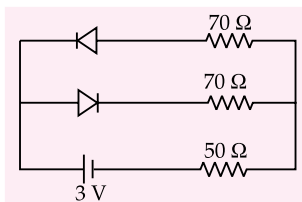
32. The electric potential between a proton and an electron is given by $V = V_0 \ln \left(\frac{r}{r_0} \right)$, where r_0 is a

constant. Assuming Bohr's model to be applicable, write variation of r_n with n , n being the principal quantum number

(a) $r_n \propto n$ (b) $r_n \propto \frac{1}{n}$
(c) $r_n \propto n^2$ (d) $r_n \propto \frac{1}{n^2}$

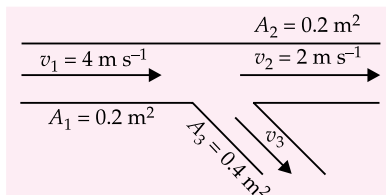
33. The circuit shown in the figure contains two diodes each with a forward resistance of 30Ω and with infinite backward resistance. If the battery is

3 V, the current through the $50\ \Omega$ resistance (in ampere) is



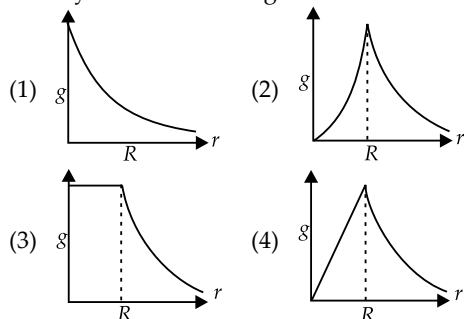
- (a) zero (b) 0.01 (c) 0.02 (d) 0.03

34. In the figure, the velocity v_3 will be



- (a) zero (b) 4 m s^{-1}
(c) 1 m s^{-1} (d) 3 m s^{-1}

35. The dependence of acceleration due to gravity g on the distance r from the centre of the earth, assumed to be a sphere of radius R of uniform density is as shown in figures below



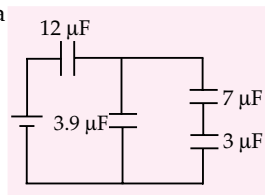
The correct figure is

- (a) (4) (b) (1) (c) (2) (d) (3)

36. A $2\ \mu\text{C}$ charge moving around a circle with a frequency of $6.25 \times 10^{12}\text{ Hz}$ produces a magnetic field 6.28 T at the centre of the circle. The radius of the circle is

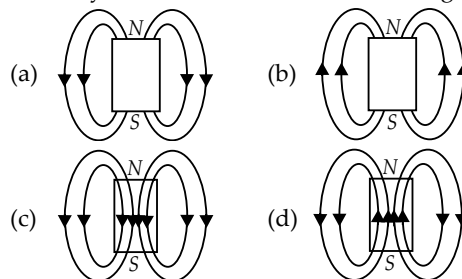
- (a) 2.25 m (b) 0.25 m (c) 13.0 m (d) 1.25 m

37. Four capacitors and a battery are connected as shown in the figure. If the potential difference across the $7\ \mu\text{F}$ capacitor is 6 V, then which of the following statements is incorrect?



- (a) The potential drop across the $12\ \mu\text{F}$ capacitor is 10 V.
(b) The charge in the $3\ \mu\text{F}$ capacitor is $42\ \mu\text{C}$.
(c) The potential drop across the $3\ \mu\text{F}$ capacitor is 10 V.
(d) The emf of the battery is 30 V.

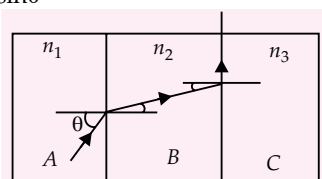
38. The magnetic field lines due to a bar magnet are correctly shown in which of the following figure.



39. A semicircular arc of radius a is charged uniformly and the charge per unit length is λ . The electric field at its centre is

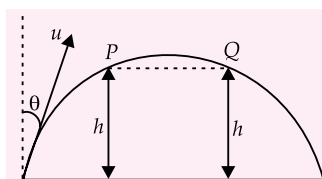
- (a) $\frac{\lambda}{2\pi\epsilon_0 a^2}$ (b) $\frac{\lambda}{4\pi\epsilon_0 a}$
(c) $\frac{\lambda^2}{2\pi\epsilon_0 a}$ (d) $\frac{\lambda}{2\pi\epsilon_0 a}$

40. A , B and C are the parallel sided transparent media of refractive index n_1 , n_2 and n_3 respectively. They are arranged as shown in the figure. A ray is incident at an angle θ on the surface of separation of A and B which is as shown in the figure. After the refraction into the medium B , the ray grazes the surface of separation of the media B and C . Then, $\sin\theta =$



- (a) $\frac{n_3}{n_1}$ (b) $\frac{n_1}{n_3}$ (c) $\frac{n_2}{n_3}$ (d) $\frac{n_1}{n_2}$

41. A particle is thrown with velocity u making an angle θ with the vertical. It just crosses the top of two poles each of height h after 1 s and 3 s respectively. The maximum height of projectile is



- (a) 9.8 m (b) 19.6 m (c) 39.2 m (d) 4.9 m

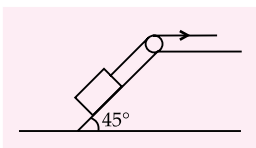
42. Two stars each of mass m and radius R approach each other to collide head-on. Initially the stars are at a distance r ($\gg R$). Assuming their speeds to be negligible at this distance of separation, the speed with which the stars collide is

(a) $\sqrt{Gm\left(\frac{1}{R} - \frac{1}{r}\right)}$ (b) $\sqrt{Gm\left(\frac{1}{2R} - \frac{1}{r}\right)}$
 (c) $\sqrt{Gm\left(\frac{1}{R} + \frac{1}{r}\right)}$ (d) $\sqrt{Gm\left(\frac{1}{2R} + \frac{1}{r}\right)}$

43. The magnetic susceptibility of a paramagnetic material at -73°C is 0.0075, its value at -173°C will be
 (a) 0.0045 (b) 0.0030
 (c) 0.015 (d) 0.0075

44. A string is stretched between fixed points separated by 75 cm. It is observed to have resonant frequencies of 420 Hz and 315 Hz. There are no other resonant frequencies between these two. Then, the lowest resonant frequency for this string is
 (a) 10.5 Hz (b) 105 Hz
 (c) 1.05 Hz (d) 1050 Hz

45. A block of mass 200 kg is being pulled up by men on an inclined plane at angle of 45° as shown. The coefficient of static friction is 0.5. Each man can only apply a maximum force of 500 N. Calculate the number of men required for the block to just start moving up the plane



- (a) 10 (b) 15 (c) 5 (d) 3

46. Water of volume 2 litre in a container is heated with a coil of 1 kW at 27°C . The lid of the container is open and energy dissipates at the rate of 160 J s^{-1} . In how much time, temperature will rise from 27°C to 77°C ?

[Given specific heat of water is 4.2 kJ kg^{-1}]

- (a) 8 min 20 s (b) 6 min 20 s
 (c) 7 min (d) 14 min

47. The waves which cannot travel in vacuum are
 (a) X-rays (b) radio-waves
 (c) infrasonic waves (d) ultraviolet rays

48. A modem is a
 (a) modulating device only
 (b) demodulating device only

- (c) modulating and demodulating device
 (d) transmitting device

49. Choose the correct statement.

- (a) A paramagnetic material tends to move from a strong magnetic field to weak magnetic field.
 (b) A magnetic material is in the paramagnetic phase below its Curie temperature.
 (c) The resultant magnetic moment in an atom of a diamagnetic substance is zero.
 (d) Typical domain size of a ferromagnetic material is 1 nm.

50. In a particular system, the unit of length, mass and time are chosen to be 10 cm, 10 g and 0.1 s respectively. The unit of force in this system will be equivalent to
 (a) 0.1 N (b) 1 N (c) 10 N (d) 100 N

SOLUTIONS

1. (a): Second overtone of closed pipe has frequency

$$= \frac{5v}{4l}$$

First overtone of open pipe has frequency

$$= 2\left(\frac{v}{2l}\right)$$

$$\therefore \frac{5v}{4l} - \frac{2v}{2l} = 100 \quad \text{or} \quad \frac{v}{4l} = 100 \quad \text{or} \quad l = \frac{v}{400}$$

Fundamental frequency of open pipe

$$= \frac{v}{2l} = \frac{v}{2\left(\frac{v}{400}\right)} = 200 \text{ s}^{-1}.$$

2. (b): Here, $\frac{R_1}{R_2} = \frac{1}{2}$ or $R_2 = 2R_1$

The equivalent resistance of parallel combination

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{R_1} + \frac{1}{2R_1} + \frac{1}{R_3} = \frac{3}{2R_1} + \frac{1}{R_3}$$

$$\text{or} \quad \frac{1}{R_3} = \frac{1}{1} - \frac{3}{2R_1} = 1 - \frac{3}{2R_1}$$

$$\text{or} \quad 1 = R_3 - \frac{3R_3}{2R_1} \quad \text{or} \quad R_3 = 1 + \frac{3R_3}{2R_1}$$

Since no resistance is in fraction, therefore minimum value of

$$\frac{R_3}{R_1} = \frac{2}{3}$$

$$\therefore R_3 = 1 + \frac{3}{2} \times \frac{2}{3} = 2 \Omega \quad \text{and} \quad R_1 = 3 \Omega$$

The maximum resistance value is

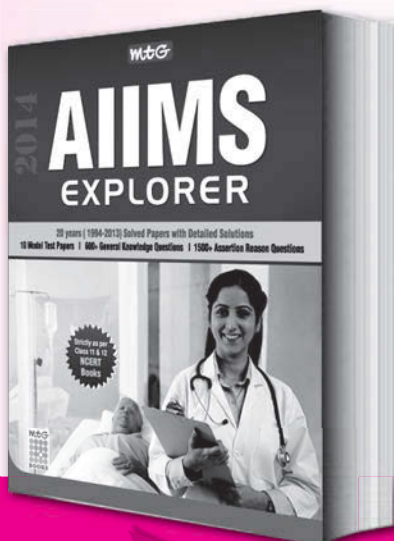
$$R_2 = 2R_1 = 2 \times 3 = 6 \Omega$$

3. (b): When stone is thrown vertically upwards from the top of tower of height h , then

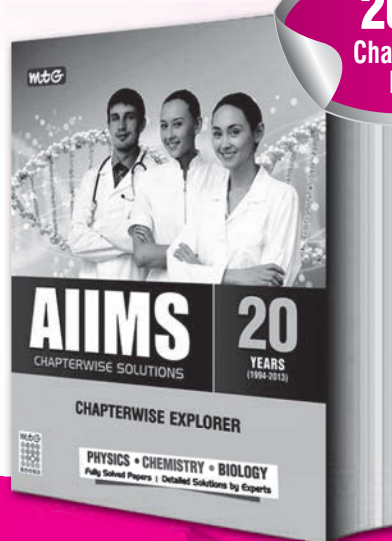


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$$h = -ut_1 + \frac{1}{2}gt_1^2 \quad \dots(i)$$

When stone is thrown vertically downwards from the top of tower, then

$$h = ut_2 + \frac{1}{2}gt_2^2 \quad \dots(ii)$$

When stone is released from the top of tower, then

$$h = \frac{1}{2}gt_3^2 \quad \dots(iii)$$

From equation (i), we get

$$\frac{h}{t_1} = -u + \frac{1}{2}gt_1 \quad \dots(iv)$$

From equation (ii), we get

$$\frac{h}{t_2} = u + \frac{1}{2}gt_2 \quad \dots(v)$$

Adding equations (iv) and (v), we get

$$h\left(\frac{1}{t_1} + \frac{1}{t_2}\right) = \frac{1}{2}g(t_1 + t_2) \text{ or } h = \frac{1}{2}gt_1t_2$$

Putting the value in equation (iii), we get

$$t_3 = \sqrt{t_1t_2}$$

4. (c) : Let P = power radiated by the sun,
 R = radius of planet

$$\text{Energy received by planet} = \frac{P}{4\pi d^2} \times \pi R^2$$

$$\text{Energy radiated by planet} = (4\pi R^2) \sigma T^4$$

For thermal equilibrium,

$$\frac{P}{4\pi d^2} \times \pi R^2 = 4\pi R^2 \sigma T^4 \Rightarrow T \propto d^{-1/2}$$

$$\therefore n = \frac{1}{2}$$

5. (b) : $\lambda = \frac{330}{500} = 0.66 \text{ m} = 66 \text{ cm}$

This tube is closed at one end. The length of the tube resonating are $\frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \frac{7\lambda}{4}$ and so on

$$\frac{\lambda}{4} = 16.5 \text{ cm}, \frac{3\lambda}{4} = 49.5 \text{ cm},$$

$$\frac{5\lambda}{4} = 82.5 \text{ cm}, \frac{7\lambda}{4} = 115.5 \text{ cm}.$$

But the 7th harmonic needs a length greater than the tube length.

\therefore There will be three resonances, as the length of the tube is only one meter.

6. (d) : The bridge $ABCD$ is balanced if

$$\frac{10}{R_1} = \frac{30}{9} \text{ or } R_1 = 3 \Omega$$

When this bridge is balanced, no current flows in arm BD . Therefore, R_2 can have any finite value.

7. (d)

8. (a) : Let f_o and f_e be the focal lengths of the objective and eyepiece respectively. For normal adjustment, distance between the objective and the eyepiece (tube length) = $f_o + f_e$. Treating the line on the objective as the object, and the eyepiece as the lens, $u = -(f_o + f_e)$ and $f = f_e$.

$$\frac{1}{v} - \frac{1}{-(f_o + f_e)} = \frac{1}{f_e}$$

$$\text{or } \frac{1}{v} = \frac{1}{f_e} - \frac{1}{f_o + f_e} = \frac{f_o}{(f_o + f_e)f_e}$$

$$\text{or } v = \frac{(f_o + f_e)f_e}{f_o}$$

$$\text{Magnification} = \left| \frac{v}{u} \right| = \frac{f_e}{f_o} = \frac{\text{image size}}{\text{object size}} = \frac{l}{L}$$

$\therefore \frac{f_o}{f_e} = \frac{L}{l}$ = magnification of telescope in normal adjustment.

9. (b) : Given $\frac{l_1}{l_2} = \frac{2}{3}$

$$\text{Resistance } R_1 = \frac{\rho l_1}{A}$$

$$R_2 = \frac{\rho l_2}{A} \therefore \frac{R_1}{R_2} = \frac{l_1}{l_2} = \frac{2}{3}$$

$$\text{As } I_1 R_1 = I_2 R_2$$

$$\therefore \frac{I_1}{I_2} = \frac{R_2}{R_1} = \frac{l_2}{l_1} \quad \dots(i)$$

$$B_1 = \frac{\mu_0 I_1 \alpha}{4\pi r} \odot \quad \dots(ii)$$

$$B_2 = \frac{\mu_0 I_2 (2\pi - \alpha)}{4\pi r} \otimes \quad \dots(iii)$$

$$\alpha = \frac{l_1}{r}; 2\pi - \alpha = \frac{l_2}{r}$$

$$\therefore B_1 = \frac{\mu_0 I_1 l_1}{4\pi r^2} \quad \dots(iv)$$

$$B_2 = \frac{\mu_0 I_2 l_2}{4\pi r^2} \quad \dots(v)$$

From (i), (iv) and (v), we get

$$B_1 = B_2$$

Hence, net magnetic field at centre $B = B_1 - B_2 = 0$

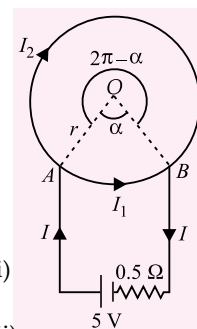
10. (b) : Total initial resistance

$$= G + R = 50 \Omega + 2950 \Omega = 3000 \Omega$$

$$\text{Current, } I = \frac{3 \text{ V}}{3000 \Omega} = 1 \times 10^{-3} \text{ A} = 1 \text{ mA}$$

If the deflection has to be reduced to 20 divisions, then current

$$I' = \frac{1 \text{ mA}}{30} \times 20 = \frac{2}{3} \text{ mA}$$



Let x be the effective resistance of the circuit, then

$$3 \text{ V} = 3000 \Omega \times 1 \text{ mA} = x \Omega \times \frac{2}{3} \text{ mA}$$

$$\text{or } x = 3000 \times 1 \times \frac{3}{2} = 4500 \Omega$$

$$\therefore \text{Resistance to be added} = (4500 \Omega - 50 \Omega) = 4450 \Omega$$

11. (c): In series LCR circuit, current is maximum at resonance.

$$\text{At resonance, } X_L = X_C \text{ or } \omega L = \frac{1}{\omega C}$$

$$\text{or } \omega^2 = \frac{1}{LC} \text{ or } L = \frac{1}{\omega^2 C}$$

$$\text{Given } \omega = 1000 \text{ s}^{-1} \text{ and } C = 10 \mu\text{F}$$

$$\therefore L = \frac{1}{1000 \times 1000 \times 10 \times 10^{-6}} = 0.1 \text{ H} = 100 \text{ mH}$$

12. (d): Here, $M = 3 \text{ kg}$, $R = 40 \text{ cm} = 0.4 \text{ m}$
Moment of inertia of the hollow cylinder about its axis is

$$I = MR^2 = 3 \text{ kg} (0.4 \text{ m})^2 = 0.48 \text{ kg m}^2$$

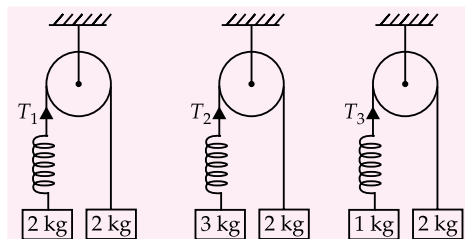
$$\text{Force applied, } F = 30 \text{ N}$$

$$\therefore \text{Torque, } \tau = F \times R = 30 \text{ N} \times 0.4 \text{ m} = 12 \text{ N m}$$

$$\text{Also } \tau = I\alpha$$

$$\therefore \alpha = \frac{\tau}{I} = \frac{12}{0.48} = 25 \text{ rad s}^{-2}$$

13. (d):



If T_1 , T_2 , T_3 are the tensions in the strings in the three cases, we have

$$T_1 = \frac{2m_1m_2g}{m_1 + m_2} = \frac{2 \times 2 \times 2g}{(2+2)} = 2g$$

$$T_2 = \frac{2 \times 3 \times 2g}{(3+2)} = 2.4g$$

$$T_3 = \frac{2 \times 1 \times 2g}{(1+2)} = 1.33g$$

$$\text{As } x \propto T \text{ and } T_2 > T_1 > T_3$$

$$\therefore x_2 > x_1 > x_3$$

14. (b): For a closed system, the total mass of gas or the number of moles remains constant.

Let n_1 and n_2 be number of moles of gas in container 1 and container 2 respectively.

$$P_1V = n_1RT_1 \text{ or } n_1 = \frac{P_1V}{RT_1} \quad \dots(i)$$

$$P_2V = n_2RT_2 \text{ or } n_2 = \frac{P_2V}{RT_2} \quad \dots(ii)$$

$$P(2V) = (n_1 + n_2)RT \quad \dots(iii)$$

Substituting the values of n_1 and n_2 in equation (iii), we get

$$P(2V) = \left(\frac{P_1V}{RT_1} + \frac{P_2V}{RT_2} \right) RT \text{ or } \frac{P}{T} = \frac{1}{2} \left(\frac{P_1}{T_1} + \frac{P_2}{T_2} \right)$$

15. (c): Here $\mu = \sqrt{2}$, $A = 30^\circ$. On reflection from mirrored surface, the ray will retrace its path, if it falls normally on the surface.

In $\triangle AED$

$$30^\circ + 90^\circ + \angle D = 180^\circ$$

$$\text{or } \angle D = 60^\circ$$

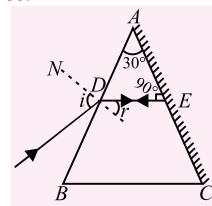
$$\text{Also } \angle D + \angle r = 90^\circ$$

$$\text{or } \angle r = 90^\circ - 60^\circ = 30^\circ$$

$$\text{As } \mu = \frac{\sin i}{\sin r}$$

$$\therefore \sin i = \mu \sin r = \sqrt{2} \sin 30^\circ = \sqrt{2} \times \frac{1}{2} = \frac{1}{\sqrt{2}}$$

$$\text{or } i = 45^\circ$$



16. (c): The photoelectric current is directly proportional to the intensity of illumination. Therefore, a change in the intensity of the incident radiation will change the photocurrent also.

17. (c): Ionisation potential of hydrogen atom is 13.6 eV. Energy required for exciting the hydrogen atom in the ground state to orbit n is given by

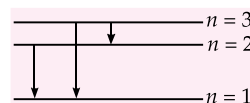
$$E = E_n - E_1$$

$$12.1 = -\frac{13.6}{n^2} - \left(-\frac{13.6}{1^2} \right) = -\frac{13.6}{n^2} + 13.6$$

$$\text{or } -1.5 = \frac{-13.6}{n^2} \text{ or } n^2 = \frac{13.6}{1.5} = 9 \text{ or } n = 3$$

Number of spectral lines emitted

$$= \frac{n(n-1)}{2} = \frac{3 \times 2}{2} = 3$$



18. (c): $\frac{h}{e^2} = \frac{[ML^2T^{-1}]}{[AT]^2} = [ML^2T^{-3}A^{-2}]$

$$\text{Resistance} = \frac{\text{Potential difference}}{\text{Current}} = \frac{[ML^2T^{-3}A^{-1}]}{[A]} = [ML^2T^{-3}A^{-2}]$$

The SI unit of resistance is ohm.

Therefore, the dimensions of ohm are same as

that of $\frac{h}{e^2}$

19. (c): Young's modulus $Y = \frac{(F/A)}{\Delta l/l}$

or $\Delta l = \frac{(F/A)l}{Y}$... (i)

Also, $\Delta l = \alpha l \Delta T$... (ii)

As per question

$$\frac{(F/A)l}{Y} = \alpha l \Delta T \text{ or } \Delta T = \frac{F}{YA\alpha}$$

$$= \frac{33000 \text{ N}}{(3 \times 10^{11} \text{ Nm}^{-2}) \times (10^{-3} \text{ m}^2) \times (1.1 \times 10^{-5} \text{ }^\circ\text{C}^{-1})}$$

$$= 10^\circ\text{C}$$

20. (d): Gravitational force of attraction on mass m at P due to solid sphere is

$$F = \frac{GMm}{(2R)^2} = \frac{GMm}{4R^2} \text{ or } \frac{GMm}{R^2} = 4F \quad \dots (i)$$

Mass of the spherical portion removed from sphere

$$M' = \frac{M}{\frac{4}{3}\pi R^3} \times \frac{4}{3}\pi \left(\frac{R}{2}\right)^3 = \frac{M}{8}$$

Gravitational force of attraction on mass m at P if mass of the spherical portion removed is present there is

$$F' = \frac{G(M/8)m}{\left(\frac{3R}{2}\right)^2} = \frac{GMm}{R^2} \times \frac{1}{8} \times \frac{4}{9}$$

$$= 4F \times \frac{1}{8} \times \frac{4}{9} = \frac{2F}{9} \quad \text{(Using (i))}$$

\therefore Gravitational force of attraction on mass m at P due to remaining part of the sphere is

$$F'' = F - F' = F - \frac{2F}{9} = \frac{7F}{9}$$

21. (d)

22. (c): Magnetic moment of the current loop is

$$\vec{m} = I\vec{A}$$

where the direction of the area vector \vec{A} , is given by right hand thumb rule. The direction of magnetic moment \vec{m} is same as the direction of the \vec{A} .

$$\therefore \vec{m}_1 = I\pi r_1^2 (-\hat{k}) = -(7 \times \pi \times 0.2^2)\hat{k} \text{ A m}^2$$

$$\text{and } \vec{m}_2 = I\pi r_2^2 \hat{k} = (7 \times \pi \times 0.3^2)\hat{k} \text{ A m}^2$$

$$\therefore \text{Net magnetic moment} = \vec{m}_1 + \vec{m}_2$$

$$= [-(7 \times \pi \times 0.2^2) + (7 \times \pi \times 0.3^2)]\hat{k} \text{ A m}^2$$

$$= [7 \times \pi \times (0.3^2 - 0.2^2)]\hat{k} \text{ A m}^2$$

$$= \left[7 \times \frac{22}{7} \times 0.05\right]\hat{k} \text{ A m}^2 = 1.1\hat{k} \text{ A m}^2$$

23. (d): Here, $m = 4 \text{ kg}$, $k = 800 \text{ N m}^{-1}$, $E = 4 \text{ J}$

$$\text{In SHM, total energy is } E = \frac{1}{2}kA^2$$

where A is the amplitude of oscillation

$$\therefore 4 = \frac{1}{2} \times 800 \times A^2$$

$$A^2 = \frac{8}{800} = \frac{1}{100}$$

$$A = \frac{1}{10} \text{ m} = 0.1 \text{ m}$$

Maximum acceleration, $a_{\max} = \omega^2 A$

$$= \frac{k}{m} A \quad \left(\because \omega = \sqrt{\frac{k}{m}}\right)$$

$$= \frac{800 \text{ N m}^{-1}}{4 \text{ kg}} \times 0.1 \text{ m} = 20 \text{ m s}^{-2}$$

24. (d): Here, $D_7 = 20 \text{ m}$, $D_9 = 24 \text{ m}$, $D_{15} = ?$

Let u and a be the initial velocity and uniform acceleration of the body.

$$D_n = u + \frac{a}{2}(2n-1) \quad \therefore D_7 = u + \frac{a}{2}(2 \times 7 - 1)$$

$$\text{or } 20 = u + \frac{13a}{2} \quad \dots (i)$$

$$\text{and } D_9 = u + \frac{a}{2}(2 \times 9 - 1)$$

$$\text{or } 24 = u + \frac{17a}{2} \quad \dots (ii)$$

Subtracting equation (i) from equation (ii), we get

$$4 = 2a \text{ or } a = 2 \text{ m s}^{-2}$$

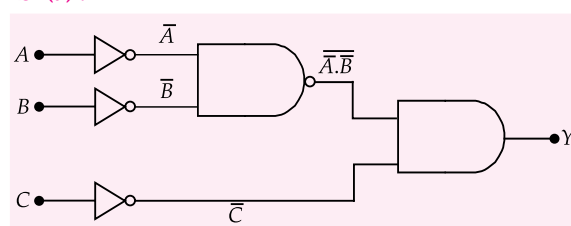
Putting this value in equation (i), we get

$$20 = u + \frac{13}{2} \times 2 = u + 13$$

$$\text{or } u = 20 - 13 = 7 \text{ m s}^{-1}$$

$$\therefore D_{15} = u + \frac{a}{2}(2 \times 15 - 1) = 7 + \frac{2}{2} \times 29 = 36 \text{ m}$$

25 (a):



$$Y = (\overline{A} \cdot \overline{B}) \cdot \overline{C} = (\overline{A+B}) \overline{C} = (A+B) \overline{C}$$

26. (d): When a source approaches a stationary observer, the frequency heard by the observer is given by

$$v = v_0 \left(\frac{v}{v - v_s} \right) \text{ where,}$$

v_0 = source frequency

v = speed of sound

v_s = speed of source

As per question

$$v_1 = \frac{v_0 \cdot 340}{(340 - 34)} \quad \text{and} \quad v_2 = \frac{v_0 \cdot 340}{(340 - 17)}$$

$$\therefore \frac{v_1}{v_2} = \frac{(340 - 17)}{(340 - 34)} = \frac{323}{306} = \frac{19}{18}$$

27. (a): Area of a square card = $1 \text{ mm} \times 1 \text{ mm} = 1 \text{ mm}^2$
Focal length of magnifying lens (converging lens),
 $f = +10 \text{ cm}$

Object distance, $u = -9 \text{ cm}$

According to thin lens formula,

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{+10 \text{ cm}} + \frac{1}{-9 \text{ cm}} = \frac{1}{10 \text{ cm}} - \frac{1}{9 \text{ cm}}$$

or $v = -90 \text{ cm}$

$$\text{Magnification, } m = \frac{v}{u} = \frac{-90 \text{ cm}}{-9 \text{ cm}} = 10$$

$$\therefore \text{ Apparent area of the card through the lens} \\ = 10 \times 10 \times 1 \text{ mm}^2 \\ = 100 \text{ mm}^2 = 1 \text{ cm}^2$$

28. (a): System A is in isobaric process

$$\therefore \Delta Q_1 = nC_p \Delta T_1$$

System B is in isochoric process

$$\Delta Q_2 = nC_v \Delta T_2$$

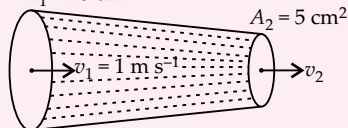
$$\therefore \Delta Q_1 = \Delta Q_2$$

$$\therefore nC_p \Delta T_1 = nC_v \Delta T_2 \quad \therefore \Delta T_2 = \frac{C_p}{C_v} \Delta T_1$$

For diatomic gas $\frac{C_p}{C_v} = \frac{7}{5}$ and $\Delta T_1 = 30 \text{ K}$

$$\therefore \Delta T_2 = \frac{7}{5} \times 30 = 42 \text{ K}$$

29. (d): $A_1 = 10 \text{ cm}^2$



According to equation of continuity,

$$A_1 v_1 = A_2 v_2$$

$$\therefore v_2 = \frac{A_1 v_1}{A_2} = \frac{10 \text{ cm}^2 \times 1 \text{ m s}^{-1}}{5 \text{ cm}^2} = 2 \text{ m s}^{-1}$$

For a horizontal pipe, according to Bernoulli's theorem

$$P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2 \Rightarrow P_2 = P_1 + \frac{1}{2} \rho (v_1^2 - v_2^2) \\ = 2000 + \frac{1}{2} \times 10^3 \times (1^2 - 2^2)$$

(\because Density of water, $\rho = 10^3 \text{ kg m}^{-3}$)

$$= 2000 - \frac{1}{2} \times 10^3 \times 3 = 2000 - 1500 = 500 \text{ Pa}$$

$$30. (d): S = \frac{t^2}{3}; \quad \frac{dS}{dt} = \frac{2t}{3}; \quad \frac{d^2S}{dt^2} = \frac{2}{3}$$

$$\text{Work done, } W = \int F dS = \int m \frac{d^2S}{dt^2} dS$$

$$W = \int m \frac{d^2S}{dt^2} \frac{dS}{dt} dt = \int_0^2 3 \times \frac{2}{3} \times \frac{2t}{3} dt = \frac{4}{3} \int_0^2 t dt$$

$$= \frac{4}{3} \int_0^2 t dt = \frac{4}{3} \left[\frac{t^2}{2} \right]_0^2 = \frac{4}{3} \times 2 = \frac{8}{3} \text{ J.}$$

31. (b): Here, $n = 500$ turns/m

$$I = 1 \text{ A}, \mu_r = 500$$

$$\text{Magnetic intensity, } H = nI = 500 \text{ m}^{-1} \times 1 \text{ A} \\ = 500 \text{ A m}^{-1}$$

As $\mu_r = 1 + \chi$

where χ is the magnetic susceptibility of the material

or $\chi = (\mu_r - 1)$

Magnetisation, $M = \chi H$

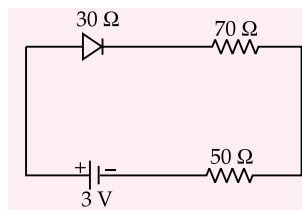
$$= (\mu_r - 1) H = (500 - 1) \times 500 \text{ A m}^{-1}$$

$$= 499 \times 500 \text{ A m}^{-1} = 2.495 \times 10^5 \text{ A m}^{-1}$$

$$\approx 2.5 \times 10^5 \text{ A m}^{-1}$$

32. (a)

33. (c): In the circuit the upper diode D_1 is reverse biased and the lower diode D_2 is forward biased. Thus there will be no current across upper diode junction. The effective circuit will be as shown in figure.

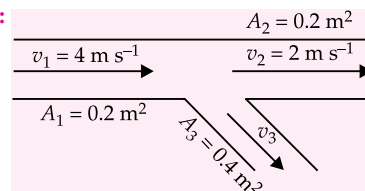


Total resistance of circuit

$$R = 50 + 70 + 30 = 150 \Omega$$

$$\text{Current in circuit, } I = \frac{V}{R} = \frac{3 \text{ V}}{150 \Omega} = 0.02 \text{ A}$$

34. (c):



According to steady flow,

$$A_1 v_1 = A_2 v_2 + A_3 v_3$$

$$\text{or } A_3 v_3 = A_1 v_1 - A_2 v_2 \text{ or } v_3 = \frac{1}{A_3} [A_1 v_1 - A_2 v_2]$$

$$= \frac{1}{0.4} [0.2 \times 4 - 0.2 \times 2] = 1 \text{ m s}^{-1}$$

35. (a)

36. (d): The charge moving on a circular orbit acts like the current loop. Magnetic field at the centre of the current loop is

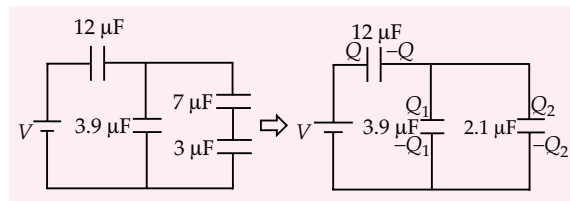
$$B = \frac{\mu_0 2\pi I}{4\pi R} = \frac{\mu_0 2\pi qv}{4\pi R} \text{ or } R = \frac{\mu_0 2\pi qv}{4\pi B}$$

Substituting the given values, we get

$$R = \frac{4\pi \times 10^{-7} \times 2\pi \times 2 \times 10^{-6} \times 6.25 \times 10^{12}}{4\pi \times 6.28}$$

$$= 1.25 \text{ m}$$

37. (c):



Equivalent capacitance of network, $C_{eq} = 4 \mu\text{F}$

Charge $Q = C_{eq}V = 4V \mu\text{C}$... (i)

The charge on the $7 \mu\text{F}$ or $3 \mu\text{F}$ capacitor

$$Q_2 = (7 \mu\text{F}) (6 \text{ V}) = 42 \mu\text{C}$$

Now,

$$\frac{Q_2}{2.1 \mu\text{F}} = \frac{Q_1}{3.9 \mu\text{F}} \Rightarrow Q_1 = (42 \mu\text{C}) \left(\frac{3.9 \mu\text{F}}{2.1 \mu\text{F}} \right) = 78 \mu\text{C}$$

$$Q = Q_1 + Q_2 = (78 \mu\text{C} + 42 \mu\text{C}) = 120 \mu\text{C} \text{ ... (ii)}$$

From equations (i) and (ii), we get

$$V = 30 \text{ V}$$

\therefore Emf of the battery, $V = 30 \text{ V}$

The potential drop across $12 \mu\text{F}$ capacitor

$$= \frac{Q}{12 \mu\text{F}} = \frac{120 \mu\text{C}}{12 \mu\text{F}} = 10 \text{ V}$$

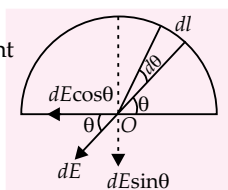
The potential drop across $3 \mu\text{F}$ capacitor

$$= \frac{Q_2}{3 \mu\text{F}} = \frac{42 \mu\text{C}}{3 \mu\text{F}} = 14 \text{ V}$$

38. (d): The magnetic field lines due to a bar magnet are closed continuous curves directed from N to S outside the magnet and directed from S to N inside the magnet. Hence option (d) is correct.

39. (d): Electric intensity at centre O, due to small element dl of charged ring

$$dE = \frac{\lambda dl}{4\pi\epsilon_0 a^2} = \frac{\lambda (a d\theta)}{4\pi\epsilon_0 a^2}$$

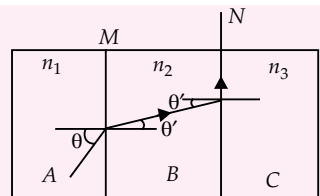


As is clear from figure, horizontal components of dE will cancel out in pairs and vertical components will add.

$$\therefore E = \int_0^\pi dE \sin \theta = \int_0^\pi \frac{\lambda}{4\pi\epsilon_0 a} \sin \theta d\theta$$

$$= \frac{\lambda}{4\pi\epsilon_0 a} [-\cos \theta]_0^\pi = \frac{2\lambda}{4\pi\epsilon_0 a} = \frac{\lambda}{2\pi\epsilon_0 a}$$

40. (a):



Applying Snell's law at M,

$$n_1 \sin \theta = n_2 \sin \theta' \text{ ... (i)}$$

Again, applying Snell's law at N

$$n_2 \sin \theta' = n_3 \sin 90^\circ$$

$$n_2 \sin \theta' = n_3$$

$$n_1 \sin \theta = n_3$$

(Using (i))

$$\sin \theta = \frac{n_3}{n_1}$$

$$41. (b): h = u \cos \theta t_1 - \frac{1}{2} g t_1^2 \text{ ... (i)}$$

$$h = u \cos \theta t_2 - \frac{1}{2} g t_2^2 \text{ ... (ii)}$$

Equating (i) and (ii), we get

$$u \cos \theta t_1 - \frac{1}{2} g t_1^2 = u \cos \theta t_2 - \frac{1}{2} g t_2^2$$

$$\text{or } u \cos \theta \times 1 - \frac{1}{2} \times 9.8 \times 1^2 = u \cos \theta \times 3 - \frac{1}{2} \times 9.8 \times 3^2$$

$$\text{or } u \cos \theta (3 - 1) = 4.9 \times (9 - 1) = 4.9 \times 8$$

$$u \cos \theta = \frac{4.9 \times 8}{2} = 4.9 \times 4 = 19.6 \text{ m s}^{-1}$$

$$\text{Maximum height} = \frac{u^2 \cos^2 \theta}{2g} = \frac{(19.6)^2}{2 \times 9.8} = 19.6 \text{ m}$$

42. (b): Since the speeds of the stars are negligible when they are at a distance of r , the initial kinetic energy of the system is zero. Therefore, initial total energy of the system is

$$E_i = \text{K.E.} + \text{P.E.} = 0 + \left(-\frac{Gmm}{r} \right) \text{ ... (i)}$$

where m represents, the mass of each star and r the initial separation between them.

When two stars collide, their centres will be at a distance twice the radius of a star i.e. $2R$. Let v is speed with which two stars collide, then total energy of the system at the instant of their collision is given by

$$E_f = \left(\frac{1}{2} mv^2 \right) \times 2 + \left(- \frac{Gmm}{2R} \right)$$

According to law of conservation of energy

$$E_f = E_i$$

$$mv^2 - \frac{Gmm}{2R} = - \frac{Gmm}{r} \text{ or } v^2 = Gm \left(\frac{1}{2R} - \frac{1}{r} \right)$$

$$\text{or } v = \sqrt{Gm \left(\frac{1}{2R} - \frac{1}{r} \right)}$$

43. (c)

44. (b) : Let the successive loops formed be p and $(p+1)$ for frequencies 315 Hz and 420 Hz.

$$\therefore v = \frac{p}{2L} \sqrt{\frac{T}{\mu}} = \frac{pv}{2L}$$

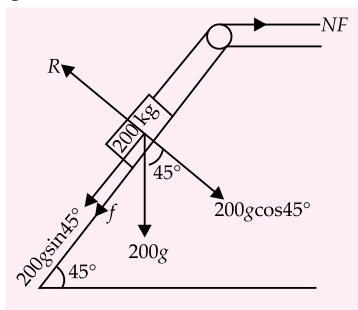
$$\therefore \frac{pv}{2L} = 315 \text{ Hz and } \frac{(p+1)v}{2L} = 420 \text{ Hz}$$

$$\text{or } \frac{(p+1)v}{2L} - \frac{pv}{2L} = 420 \text{ Hz} - 315 \text{ Hz}$$

$$\text{or } \frac{v}{2L} = 105 \text{ Hz} \Rightarrow \frac{1 \times v}{2L} = 105 \text{ Hz}$$

$p = 1$ for fundamental mode of vibration of string.

45. (c) :



Here, mass of the block, $m = 200 \text{ kg}$

coefficient of static friction, $\mu_s = 0.5 = \frac{1}{2}$

angle of incline plane, $\theta = 45^\circ$

Maximum force that each man can apply, $F = 500 \text{ N}$

Let N number of men are required for the block to just start moving up the plane

$$\begin{aligned} NF &= mg \sin \theta + f \\ &= mg \sin \theta + \mu_s R \\ &= mg \sin \theta + \mu_s mg \cos \theta \\ &= mg [\sin \theta + \mu_s \cos \theta] \\ &= 200 \times 10 \left[\sin 45^\circ + \frac{1}{2} \cos 45^\circ \right] \end{aligned}$$

$$= 200 \times 10 \left[\frac{1}{\sqrt{2}} + \frac{1}{2\sqrt{2}} \right] = \frac{200 \times 10 \times 3}{2\sqrt{2}}$$

$$N = \frac{200 \times 10 \times 3}{2\sqrt{2} \times 500} \approx 5$$

46. (a) : Using law of conservation of energy, energy produced by heater = heat gained by water + energy lost

$\therefore Pt = ms\Delta T + \text{energy lost}$

$$1000t = 2 \times (4.2 \times 10^3) \times (77 - 27) + 160t$$

On solving, we get $t = 500 \text{ s} = 8 \text{ min } 20 \text{ s}$

47. (c) : X-rays, radiowaves and ultraviolet rays are electromagnetic waves and do not require a medium to travel whereas infrasonic are mechanical waves and they require a medium to travel. Hence infrasonic waves do not travel in vacuum.

48. (c) : Modem performs the functions of both the modulator and the demodulator. Modem acts as a modulator in the transmitting mode and it acts as a demodulator in the receiving mode.

49. (c) : Diamagnetic substances are those substances in which resultant magnetic moment in an atom is zero.

A paramagnetic material tends to move from a weak magnetic field to strong magnetic field.

A magnetic material is in the paramagnetic phase above its Curie temperature.

Typical domain size of a ferromagnetic material is 1 mm.

50. (a) :

$$\begin{array}{ll} M_1 = 10 \text{ g} & M_2 = 1 \text{ kg} \\ L_1 = 10 \text{ cm} & L_2 = 1 \text{ m} \\ T_1 = 0.1 \text{ s} & T_2 = 1 \text{ s} \\ n_1 = 1 & n_2 = ? \end{array}$$

$$n_2 = n_1 \left(\frac{M_1}{M_2} \right)^a \left(\frac{L_1}{L_2} \right)^b \left(\frac{T_1}{T_2} \right)^c$$

The dimensional formula of force is $[MLT^{-2}]$

$$\therefore a = 1, b = 1, c = -2$$

$$= 1 \left(\frac{10 \text{ g}}{1 \text{ kg}} \right)^1 \left(\frac{10 \text{ cm}}{1 \text{ m}} \right)^1 \left(\frac{0.1 \text{ s}}{1 \text{ s}} \right)^{-2}$$

$$= 1 \left(\frac{10^{-2} \text{ kg}}{1 \text{ kg}} \right)^1 \left(\frac{10^{-1} \text{ m}}{1 \text{ m}} \right)^1 \left(\frac{0.1 \text{ s}}{1 \text{ s}} \right)^{-2}$$

$$= \frac{1 \times 10^{-2} \times 10^{-1}}{10^{-2}} = 10^{-1} = 0.1$$

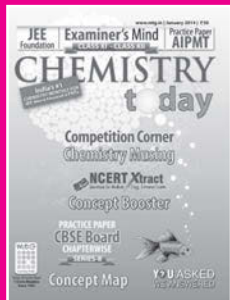
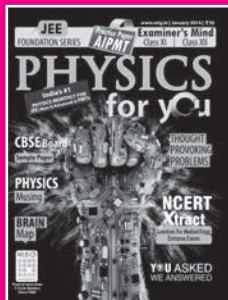
Hence, the unit of force in a given system will be equivalent to 0.1 N.

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